

Low Latency Wireless Ad-Hoc Networking: Power and Bandwidth Challenges and a Hierarchical Solution

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Abstract

This paper is concerned with the scaling of the number of hops in a large scale wireless ad-hoc network (WANET), a quantity we call network latency. A large network latency affects all aspects of data communication in a WANET, including an increase in delay, packet loss, required processing power and memory. We consider network management and data routing challenges in WANETs with scalable network latency. On the physical side, reducing network latency imposes a significantly higher power and bandwidth demand on nodes, as is reflected in a set of new bounds. On the protocol front, designing distributed routing protocols that can guarantee the delivery of data packets within scalable number of hops is a challenging task. To solve this, we introduce multi-resolution randomized hierarchy (MRRH), a novel power and bandwidth efficient WANET protocol with scalable network latency. MRRH uses a randomized algorithm for building and maintaining a random hierarchical network topology, which together with the proposed routing algorithm can guarantee efficient delivery of data packets in the wireless network. For a network of size N , MRRH can provide an average latency of only $O(\log^3 N)$. The power and bandwidth consumption of MRRH are shown to be *nearly* optimal for the latency it provides. Therefore, MRRH, is a provably efficient candidate for truly large scale wireless ad-hoc networking.

I. INTRODUCTION

Efficient data delivery in wireless ad-hoc networks (WANET's) is a challenging task due to the lack of any global coordination of the nodes in the network. Network management protocols, as well as routing algorithms have to work in an autonomous manner, and yet overall, the network should be able to efficiently route traffic from any source to any destination. Most routing strategies are also interested in minimizing the average power required for network operation, which requires the use of short-distance communication. The ease of routing and power considerations are, perhaps, the primary reasons why most known WANET routing protocols resort to simple, Nearest-Neighbor Communication (NNC) strategies and their variants, for routing.

Nearest neighbor communication, however, can not scale properly as the system size grows, and WANETs of increasingly larger sizes become realizable. In its simplest form, for a network of N nodes, the NNC strategy will use $\sim \sqrt{N}$ relay nodes for an average communication. We call *the number of hops a WANET uses for an average communication*, its **network latency** (NL). The name is chosen because this form of latency is induced by the network layer, and is a reflection of the underlying routing algorithm¹. The average network latency of $\Theta(\sqrt{N})$ could significantly impact the system performance of NNC

¹Note that network latency, unlike delay, is independent of the communication rate. Delay, on the other hand, is usually defined as the reciprocal of communication rate (or throughput), and is a measure of the amount of time required to communicate a given volume of data between two points.

based routing schemes, as summarized below.

(1) Increasing the NL will increase the communication delay due to buffering at relay nodes. In steady state, one expects the average *delay to be linearly proportional to the NL*.

(2) As NL increases, the probability of end-to-end packet loss increases. This may require excessive number of packet retransmissions, which in turn would increase average power consumption. Such tradeoffs have recently received considerable attention (see e.g., [19], [20], [21], [22]).

(3) An increase in NL induces computational and memory overhead on a larger number of nodes in the network. In a nearest neighbor communication scheme, e.g., a node has to process and route one packet out of every $\Theta(\sqrt{N})$ packets transmitted in the network which directly translates into a need for more processing power.

Therefore, NNC is most suited for small to medium size applications.

New routing schemes with significantly better NL performance have to be devised for the WANETs, if they want to scale to truly large sizes; perhaps with millions of participants. Designing a large-scale low-NL WANET, however, brings about many physical as well as algorithmic challenges that are considered next. This paper focuses on wireless ad-hoc networking schemes with low network latency, and the effect of reducing the number of communication hops on critical performance measures of the networks, including power, bandwidth and throughput.

A. Low Network-Latency WANET's: Challenges

Distributed operation of a WANET becomes a significant challenge when NL has to be low, i.e., packets have to be delivered within only a small number of hops. The reason is that a node has to expect data packets from other nodes that are far away in the network. This is unlike NNC models where a node has to manage its communication with only a limited number of spatially close by nodes. As described next, scheduling routes, multiple access control (e.g., time and frequency division multiplexing), as well as, code management become particularly nontrivial in a low NL WANET.

To be specific, lets consider the hypothetical and extreme scenario of single hop or direct communication. Suppose also that the bandwidth and power requirements are not issues, and thus, to get the shortest possible NL (i.e., one), any source node directly communicates with its destination node. To eliminate interference, each communication pair can be assigned an independent non-interfering channel with enough bandwidth. Assume that at any round, every node is communicating with another, randomly chosen, node. For a network of size N , therefore, a total of at least $N/2$ communication channels have to be set up for each round of communication; something that might be prohibitive even for moderately large size N . Now, in the next round, each node will decide to change its communication partner. How would a given node notify its destination node about the channel in which the communication has to take place? How does the network, in a decentralized way, schedule and assign channels to communication pairs? This task seems impossible, unless there is one dedicated channel for every pair of nodes (a total

of $N(N - 1)/2$ channels), which is certainly impractical. Even then, this requires every node to have global information about every other node in the network.

None of the above problems would have been an issue in an NNC strategy. There, a node is only responsible for receiving and relaying packets to a constant number of close neighbors. Multiple access control (MAC) can be performed by local negotiations, and one of the many local routing algorithms could have been adopted, a few of which will be reviewed later in this section.

Since single-hop communication is impractical in ad-hoc situation (*even if* power and bandwidth are no constraints), we then ask the question, whether distributed protocols exist for efficient joint multiple access control and data routing (similar to NNC), while providing a low NL (similar to direct communication)? This paper proposes one such solution, Multi-Resolution Randomized Hierarchy (MRRH), based on randomized data structure concepts.

Apart from MAC and routing challenges, low NL communication is demanding in terms of bandwidth and power requirements. Reducing the number of hops requires the use of longer communication links, which in turn requires more power. Increasing the power for a communication link, on the other hand, will increase the interference on other nodes in the network, which calls for an increased bandwidth. Any proposed low NL networking protocol has to be evaluated with respect to these tradeoffs. To quantify this, *we derive a new set of fundamental tradeoffs* among NL, node density, throughput, average power, number of channels and total bandwidth of any WANET. These bounds quantify the tradeoffs between various performance measures of a WANET. Our proposed scheme, MRRH, is then shown to nearly satisfy these bounds; in other words, *power and bandwidth requirement of MRRH is nearly optimal given the NL it provides.*

B. Performance Bounds and Proposed Algorithm

Our performance bounds relate NL to different metrics of a WANET and are derived in Section III. As shown in Theorem 3.2, for a direct line-of-sight power dissipation, the average power consumption of any WANET with NL of L should scale as $\Omega(N/L^2)$, where N is the network size, regardless of the bandwidth. Similarly, Theorem 3.3 states that the total bandwidth required by any WANET with NL of L should again scale as $\Omega(N/L^2)$. To keep the NL almost constant (independent of N), one should be prepared to scale the average power and bandwidth at least linearly with the network size.

The above bounds lead to the question of designing efficient decentralized routing and MAC protocols. We propose Multi-Resolution Randomized Hierarchy (MRRH), a novel randomized algorithm for efficient wireless ad-hoc networking and measure various performance metrics of MRRH. Nodes in an MRRH network manage their communication strategies locally. Even more interestingly, *network evolution in MRRH is totally stateless*, that is, nodes randomly and independently change their strategies regardless of all other nodes in the network, making MRRH a perfect candidate for mobile environments. The proposed *routing algorithm defined on the underlying random network is also stateless*: A node will decide on where to relay a received packet based only on its own position, the position of the target,

and the positions of at most $O(\log N)$ neighboring nodes. Even though the routing is stateless, the NL of MRRH, in terms of the average number of hops per communication, is only $O(\log^3 N)$. Furthermore, the power and bandwidth requirement of MRRH are nearly optimal among all routing algorithms that have an average NL equal to the one provided by MRRH, as is explained next.

The main idea behind MRRH is to superimpose several virtual topologies to form a nested hierarchical structure. At all times, nodes that belong to a higher hierarchy are also members of all lower hierarchies. The average distance of communication in upper hierarchies is exponentially longer than the ones in lower hierarchies. The fraction of times a node is part of a higher hierarchy, however, is exponentially smaller. A packet is usually relayed starting from a low hierarchy. If the target node is far away, then the packet automatically *climbs up* the hierarchy and quickly reaches a node in upper hierarchies. These hierarchical structures are not constant, and will change from time to time or even from packet to packet. Nodes that operate in higher hierarchies for some routes might be part of lower hierarchies for others. This will provide a natural load balance to the system. The overall algorithm however, guarantees correct delivery of all packets even in this highly changing environment.

C. Relation to Previous Work

Our routing algorithm is based on position. Various position-based routing algorithms have been proposed for WANET's. For the purpose of this paper, these algorithms can be divided into two main categories [1]:

(i) Nearest neighbor, approximate line of sight, routing:

Algorithms based on nearest neighbor communication pass on a data packet to a close by node which is closer to the destination. Various variations on this theme can be found in [3], [4], [5], [6], [15], [16] and many other papers. Nearest neighbor communications incurs a large NL, often inappropriate for large scale operations. In a network of size N with nodes randomly distributed on a square, nearest neighbor communication requires an $\Theta(\sqrt{N})$ hops for an average communication. To reduce the NL, various hierarchical routing algorithms have been considered.

(ii) Hierarchical routing algorithms:

In these algorithms, the routing is done in (usually two) different levels. For instance, zone-based routing algorithms divide the network to various zones. The routing is divided into two steps, routing between the zones, and routing within the zones by introducing a set of “dominating nodes”. A dominating node is able to reach any node within a zone and also is capable of communicating with dominating nodes in other zones. Variations of this idea have been proposed in many papers including [7], [8], [9], [10], [11]. Since there are fewer dominating nodes compared to all nodes, the number of hops for an average communication reduces. While these two level hierarchical schemes mitigate the NL problem to some extent in medium size applications, they will not scale appropriately to extremely large network sizes.

MRRH too is a hierarchical system, except that the number of hierarchal levels is not constant, and that nodes frequently join and leave hierarchies through local decisions. The overall design of MRRH

however, ensures that, (i), the network is totally connected within each level of hierarchy at all times, and (ii) there is always a polylogarithmically ² small path between any pair of nodes, that might pass through various hierarchical levels, and (iii) this path can be discovered locally through a simple greedy algorithm. A greedy routing algorithm will then be able to efficiently route data packets from any source to any destination within small latencies.

The rest of this paper is organized as follows. In Section II, we introduce the problem model. In Section III, a set of new general constraints are derived that relate the maximum feasible throughput to the average power, bandwidth and NL. MRRH along with our proposed routing algorithm is introduced in Section IV. Then in Section V, power and bandwidth requirement of MRRH are derived. Comparison with results in Section III enables us to prove that MRRH is nearly optimal in its bandwidth and power usage. Section VI provides concluding remarks.

II. PRELIMINARIES

This section formally introduces the WANET model used in this paper, including the network topology, traffic and communication models.

A. Network Model

The wireless network model considered in this paper consists of a set of N nodes, randomly distributed on the surface of a sphere of radius R and area $A = 4\pi R^2$. We denote the set of all nodes by Γ . Symmetric surface of the sphere simplifies geometric proofs of the paper. Most of the proofs however can apply to a regular two dimensional geometry (e.g., a square) with only simple modifications.

Each node can act both as a transmitter and a receiver. Every node has access to K non-interfering Additive White Gaussian Noise (AWGN) channels. The bandwidth of channel $k \in \mathcal{K} \triangleq \{0, 1, 2, \dots, K-1\}$ is assumed to be B^k and the noise power spectral density is η_0 for all channels. At each channel and at any point of time, a node is assumed to be transmitting data to or receiving data from at most one other node. Therefore, time sharing has to be used for communication to multiple nodes over a single channel. For simplicity, it is assumed that the communication at each channel is performed with a common power P^k , i.e., if a node decides to transmit on channel k , it will do so with a power P^k . The participation function, $\phi_t^k(i, j)$, is one if node i is transmitting to node j over its k^{th} channel at time t and is zero otherwise. With this definition, at a given time t , a node i is assumed to be able to communicate to a node j with rate $R_t^k(i, j)$ equal to the capacity of the corresponding AWGN channel:

$$R_t^k(i, j) = B_k \log_2 \left(1 + \frac{P^k \gamma(\|X_i - X_j\|)}{B_k \eta_0 + \sum_{l \neq j} P^k \phi_t^k(j, l) \gamma(\|X_j - X_l\|)} \right)$$

where X_x is the position of a node x and $\|\cdot\|$ is the geodesic distance on the surface of the sphere and $\gamma : \mathbb{R}^+ \rightarrow (0, 1]$ is a power dissipation function.

²A function $f(N)$ is said to be polylogarithmic, if there exist $m, M > 0$ such that $f(N) < \log^m(N)$ for all $N > M$.

Average power consumption at a node i is:

$$P_{avg}(i) = E_t \left\{ \sum_{j \in \Gamma} \phi_t^k(i, j) P_t^k(i, j) \right\} \quad (1)$$

where $E_t\{\cdot\}$ denotes time averaging. $P_{avg} = N^{-1} \sum_{i \in \Gamma} P_{avg}(i)$ is the total average power. The total bandwidth of the system is $B = \sum_{k=0}^{K-1} B^k$.

For a fixed P_{avg} and B , the choice of the power and bandwidth levels P^k, B^k are left to the WANET designer.

B. Multi-Hop Routing

Data routing is performed through a decentralized multi-hop algorithm. Upon receiving the data packets for a destination node, a relay node should be able to locally decide on where to send the packet next. Ideally, the routing algorithm should be stateless, that is, (1) the data packet should only contain information about the destination and possibly the source node and (2) routing decisions should be made on a ‘‘per packet’’ basis. In our model we assume nodes are equipped with Global Position Systems (GPS) and addressing is by position, i.e., packets contain the position of the destination node. A routing algorithm is said to have an average NL of L if each packet, on average, has to be relayed L times before it reaches the destination, where the averaging is done over nodes and time.

C. Traffic Model

We assume a uniform and symmetric traffic model. At any given time, any node i is sending packets to exactly one node j at a rate of λ bits per second, called the throughput. We call a throughput λ feasible if it can be relayed successfully at all nodes using a finite buffer size.

Equivalently, a throughput λ is feasible with a participation policy function $\phi_t^k(i, j)$ if and only if:

$$\forall i \in \Gamma, \quad \sum_{k \in \mathcal{K}} \left(E_t \left\{ \sum_{j \in \Gamma} \phi_t^k(i, j) R_t^k(i, j) \right\} - E_t \left\{ \sum_{l \in \Gamma} \phi_t^k(l, i) R_t^k(l, i) \right\} \right) \geq 0$$

where E_t denotes expectation over time, and we have considered the fact that every node is a sink of data with rate λ and the source of some data with the same rate λ .

In this paper, we will be interested in minimizing the average NL L while maximizing a feasible throughput λ and minimizing the average power consumption P_{avg} and bandwidth requirement B . These requirements are of course conflicting. A set of bounds are derived in Section III that quantifies these conflicts. We then propose a system capable of efficiently trading off these conflicting figures of merits. The proposed system, called Multi-Resolution Randomized Hierarchy (MRRH) is introduced in Section (IV) and is shown to be nearly optimal in the light of the bounds derived in Section III.

III. POWER AND BANDWIDTH REQUIREMENT OF LOW NL WANETS

In this section, we derive two new lower bounds on the average power and total bandwidth requirement of any WANET with a feasible constant throughput of λ and average NL of L .

A. A Lower Bound on Average Power

Reducing the average NL requires and increase in the average communication length which imposes a constraint on the average power required for communication. We quantify this requirement in this subsection.

We first prove the following simple Lemma.

Lemma 3.1: On average, $N/16$ of all communication pairs are between nodes that are at least a distance $R/4$ apart. Therefore, at least an average of $\lambda/16$ bits per second should be communicated between nodes that are at least a distance $R/4$ apart.

Proof:

This is easily proved as follows. Consider two caps on the poles of sphere with angles $\theta_1 \in (\pi/4, \pi/2)$ and $\theta_2 \in (-\pi/2, -\pi/4)$. Each of these caps has an area $R^2\sqrt{2}/2 \geq A/4$ and therefore contains on average $N/4$ nodes. The probability that two nodes, one each cap, communicate in any given round is therefore at least $(1/4)^2 = 1/16$. The minimum distance between these two nodes is $R/4$. ■

Let Γ_t^T, Γ_t^R indicate the set of nodes that are transmitting to and receiving data from other nodes that are at least $R/4$ away. We have shown that $E_t\{|\Gamma_t^T|\} = E_t\{|\Gamma_t^R|\} \geq N/16$. Note that when the communication latency is at most L , each packet starting from a node in Γ_t^T and ending in a node in Γ_t^R must pass over a link that is at least $R/(4L)$ long.

Lemma 3.1 leads to the following Theorem on average power consumption of any MANET with average NL L .

Theorem 3.2: Any routing algorithm with feasible throughput λ and average NL L requires an average power of at least $P_{avg} > \frac{\eta_0\lambda}{16\gamma(\frac{R}{4L})\ln 2}$. For a physical power dissipation ($\gamma(D) \propto D^{-2}$), and the node density a constant ρ , one has $P_{avg} \geq \lambda\rho^{-1}\eta_0^{-1}48^{-1}(4\pi)^{-1}NL^{-2}$.

Proof: It is easy to show that:

$$R_t^k(i, j) \leq \ln 2 P^k \gamma(\|X_i - X_j\|) \phi_t^k(i, j) / \eta_0 \quad (2)$$

by letting $B^k \rightarrow \infty$ in Eqn. (1). Consider a data stream of rate r_{xy} bits per second, starting from a node $x \in \Gamma_t^T$ that reaches a node $y \in \Gamma_t^R$ using only L steps. Data packets have to be communicated over a link that is at least $R/(4L)$ long. This requires a power of at least $P_{xy} \geq r_{xy}\eta_0 \ln 2 / \gamma(R/(4L))$. The total power required for accommodating all communications between nodes in Γ_t^T, Γ_t^R only is at least:

$$\begin{aligned} P_{avg}N &\geq \sum_{x \in \Gamma_t^T, y \in \Gamma_t^R} P_{xy} \\ &\geq \frac{\eta_0}{\gamma(R/(4L)) \ln 2} \sum_{x \in \Gamma_t^T, y \in \Gamma_t^R} r_{xy} \\ &\geq \frac{\eta_0 N \lambda}{16\gamma(\frac{R}{4L}) \ln 2} \end{aligned}$$

which proves the claim. ■

B. Lower Bound on Total Bandwidth

Theorem (3.2) proved that for a physical system with almost constant NL L , the average power should scale at least linearly with N . We next show that a system with low NL requires a considerable amount of bandwidth to scale.

The idea is that , from Lemma 3.1, to provide a low NL, an average communication has to use long range communication on one of the various channels. A long range communication in a channel will significantly interfere with most of the nodes in the same channel , disabling them from simultaneous communication. This in turn limits the amount of data that can be mobilized in the network. This is quantizes in the following Theorem.

Theorem 3.3: For any system with average NL of L and a feasible throughput of λ one requires that:

$$B \geq \frac{\gamma(2\pi R)}{\gamma(\frac{R}{4L})} \times \frac{N\lambda}{16K\eta_0}$$

Proof:

Consider B_k the bandwidth of some channel k . At the time t , let's define $P_R(i)$ as the signal power received by node i over channel k corresponding to a long range communication only. If i dose not communicate in k at t , that is, if $i \notin \Gamma_t^k$, then $P_R(i) = 0$. Similarly, let $P_I(i)$ represent the interference of all other long range communications at node i at time t .

Let $\mathcal{R}_k(i)$ denote the average communication rate a node i receives through long range communication. Then:

$$\begin{aligned} \mathcal{R}_k(i) &\leq B_k \log \left(1 + \frac{P_R(i)}{B_k \eta_0 + P_I(i)} \right) \\ &\leq \frac{B_k P_R(i)}{\eta_0 B_k + P_I(i)} \\ &\leq \frac{B_k P_R(i)}{\eta_0 B_k + \sum_{j \neq i} \frac{\gamma(2\pi R)}{\gamma(R/(4L))} P_R(j)} \\ &= B_k c^{-1} \frac{c P_R(i)}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} \end{aligned} \quad (3)$$

where we have defined $c \triangleq \frac{\gamma(2\pi R)}{\gamma(R/(4L))}$. The last inequality is understood as follows: suppose node j receives a power P corresponding to a long range communication from a node j . By definition, node j is at least $R/(4L)$ apart from i . Therefore, the power is has used for communication must be at least $P/\gamma(\frac{R}{4L})$. This power imposes at least $\gamma(4\pi R)P/\gamma(\frac{R}{4L})$ interference on all nodes other than i .

The total communication rate over long range connections is bounded as:

$$\sum_i \mathcal{R}_k(i) \leq B_k c^{-1} \sum_i \frac{c P_R(i)}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} \quad (4)$$

The right hand side of the above inequality can be maximized by noting that:

$$\begin{aligned}
H &\triangleq \sum_i \frac{cP_R(i)}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} \\
&= \sum_i \left(\frac{cP_R(i) + \eta_0 B_k + c \sum_{j \neq i} P_R(j)}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} - 1 \right) \\
&= \sum_i \frac{\eta_0 B_k + c \sum_j P_R(j)}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} - N \\
&= \sum_i \frac{\eta_0 B_k + cT_R}{\eta_0 B_k + c \sum_{j \neq i} P_R(j)} - N
\end{aligned}$$

where T_R is the total received signal power used for long range communication.

Now let's make the following change of variable: $Q(i) = \sum_{j \neq i} P_R(j)$. Then, :

$$H = \sum_i \frac{\eta_0 B_k + cT_R}{\eta_0 B_k + cQ(i)} - N \quad (5)$$

Now note that $\sum_i Q(i) = (N-1) \sum_i P_R(i) = (N-1)T_R$. For a fixed T_R , maximizing (5) under this constraint will require $Q(i) = (N-1)T_R/N$, which results in:

$$\begin{aligned}
H &\leq N \frac{\eta_0 B_k + cT_R}{\eta_0 B_k + c \frac{(N-1)}{N} T_R} - N \\
&= N \left(\frac{\eta_0 B_k + cT_R}{\eta_0 B_k + c \frac{(N-1)}{N} T_R} - 1 \right) \\
&= N \left(\frac{c \frac{T_R}{N}}{\eta_0 B_k + c \frac{(N-1)}{N} T_R} \right) \\
&\approx c \frac{T_R}{\eta_0 B_k + cT_R} \quad (6)
\end{aligned}$$

Inserting this back into (4):

$$\sum_i \mathcal{R}_k(i) \leq \frac{B_k T_R}{\eta_0 B_k + cT_R}$$

The above inequality holds for any channel k . We know that the rate of communication over long range connections should be at least $N\lambda/16$ (Lemma 3.1). Therefore, we need to have $\sum_{k \in \mathcal{K}} \sum_i \mathcal{R}_k(i) \geq (N\lambda)/(16\eta_0 \ln 2)$. Therefore, there should exist at least one channel k^* such that,

$$\sum_i \mathcal{R}_{k^*}(i) \geq (N\lambda)/(16\eta_0 \ln 2)/K$$

For this channel,

$$\frac{cB_{k^*} T_R^*}{\eta_0 B_{k^*} + cT_R^*} \geq c(N\lambda)/(16\eta_0)$$

Noting that all components are positive, the necessary condition for the above inequality to hold is that:

$$T_R^* \geq (N\lambda)/(16K\eta_0 \ln 2)$$

and that

$$B_{k^*} \geq c(N\lambda)/(16K\eta_0) = \frac{\gamma(2\pi R)}{\gamma(\frac{R}{4L})} \frac{N\lambda}{16K\eta_0 \ln 2}$$

■

The next section introduces our low NL MRRH solution. We provide an algorithmic description of MRRH, prove its correctness and NL properties and calculate its power and bandwidth requirements. Equipped with the results in this section, we will be able to show that MRRH requires an almost optimal power and bandwidth among all systems that provide the same average NL as MRRH does.

IV. MULTI-RESOLUTION RANDOMIZED HIERARCHY

Multi-Resolution Randomized Hierarchy (MRRH) is a joint participation policy and routing algorithm for efficient delivery of data packets, as follows.

MRRH uses $K = \log N - 2 \log \log N$ different channels, where logs are all in base 2 unless otherwise specified. Let $\angle(i, j)$ denote the spherical angle between two nodes i, j . Take a cap on the sphere with spherical angle θ_k and surface area $A_k = 2\pi R^2(1 - \cos \theta_k)$ such that $A/A_k = (16 \log N)^{-1} 2^{-k} N$, where $A = 4\pi R^2$ is the total surface of the sphere. Caps of angle θ_k will determine the communication neighborhood of MRRH at channel k . In other words, two nodes $i, j \in \Gamma$ will communicate in channel k only if $\angle(i, j) < \theta_k$ in which case we say i, j are neighbors in the k^{th} channel.

We call A_k the coverage area of channel k . The coverage area as well as the communication range grow exponentially with the channel level k . For simplicity, we define the set of neighbors of i in the k^{th} channel as $\Psi^k(i)$.

Note that, $A_k = 2\pi R^2(1 - \cos \theta_k) \leq 2\pi R^2 \theta_k^2$. Thus, for any two nodes i, j that are *not* neighbors in channel k we have:

$$\begin{aligned} \|X_i - X_j\| &\geq 2\pi R \theta_k \geq 2\pi \sqrt{\frac{A_k}{2\pi R^2}} \\ &= \sqrt{2\pi A_k} \\ &= \sqrt{2\pi \cdot 4\pi (16 \log N) 2^k N^{-1}} \\ &= 8\pi R \sqrt{2^k N^{-1} \log N} \end{aligned} \tag{7}$$

We will use the above fact to upper bound the amount of interference a node's communication incurs on other nodes that are not in its neighborhood.

The participation policy of MRRH, $\phi_i^k(i, j)$, is randomized. Each node i will randomly choose a value $0 \leq l(i) \leq K$, called its level, as follows: $Pr\{l(i) = 0\} = 1/2$ and $Pr\{l(i) = k+1\} = Pr\{l(i) = k\}/2$ for $0 < k < K-1$. For $k = K$, the probability is $Pr\{l(i) = K\} = 1/2 + (1/2)^{K-1}$. Each node i with

level $l(i)$ will only open its first $l(i)$ channels, i.e., it will only communicate on channels $k = 0, 1, \dots, l(i)$. Note that if channel k of any node i is open, all lower channels $(0, 1, \dots, k - 1)$ of i are also open.

Evidently, the above participation policy is a homogenous one, i.e., it treats all the nodes uniformly, but imposes a heterogenous structure by placing nodes randomly at different levels (or hierarchies). The density of the nodes decreases exponentially as their level increases. As will be seen shortly, nodes in upper levels will have to spend an exponentially larger amount of power.

When the nodes in the WANET have heterogenous power capabilities, the level of nodes can be aligned with those capabilities. In other words, nodes can choose a level between 0 and K based on their power capabilities. All the results in this section remain unchanged provided that the density of nodes in different levels follows the same exponential relation. For clarity, however, we state all results for a homogenous setting.

The routing is greedy. When a node i receives a packet destined for a node j , it checks the neighborhood of all its *open* channels and sends the packet to a neighbor that is closest to the destination node j and is also closer to j than i itself. If no such node exists, the routing is stopped. The routing either stops at the target node j (successful routing) or at a wrong node $j' \neq j$ (unsuccessful routing).

The reader familiar with data structures and algorithms has probably noticed the similarity of MRRH with SkipLists [17]. SkipLists are randomized versions of binary search trees. MRRH has many similarities to SkipLists in its hierarchical structure and greedy search strategy. SkipLists however do not have any notion of geographical location, dimension, power, bandwidth and interference.

This section will prove various routability properties of MRRH. These routing properties are geometric in nature and do not take into account physical requirements of routing (i.e., rate, power and bandwidth requirements). Rather, they are only concerned with proving the correctness of the data delivery and calculating the average and maximum NL. Following sections, on the other hand, will find the power and bandwidth required by MRRH for making a desired throughput feasible. There, for instance, we will employ a simple Time Division Multiple Access (TDMA) scheme for shared channel access by neighboring nodes.

Throughout, an event is said to happen with high probability (w.h.p) if it happens with probability at least $1 - N^{-2}$. The correctness of routing for MRRH can be proved by methods close to continuum percolation arguments [13], [14]. We adopt a different approach for proving this. Taking any pair of nodes i, j , we will show that there exists a chain of nodes $s_1 = i \rightarrow s_2 \rightarrow \dots \rightarrow s_m = j$ such that $s_{n+1} \in \Psi^k(s_n)$ and $\angle(s_{n+1}, j) < \angle(s_n, j) - \epsilon_k$, for some constant $\epsilon_k > 0$. In other words, there is a chain of neighbors that will take any packet starting from i to j and the packet gets strictly closer to j at each step (Lemma 4.3).

For that we need a number of simple lemmas.

Lemma 4.1: Any surface of area $S = 2^{k-2}A_0$ w.h.p. contains at least $2 \log N$ nodes and at most $6 \log N$.

Proof: This follows easily from Chernoff bound. Note that the average number of nodes in S is

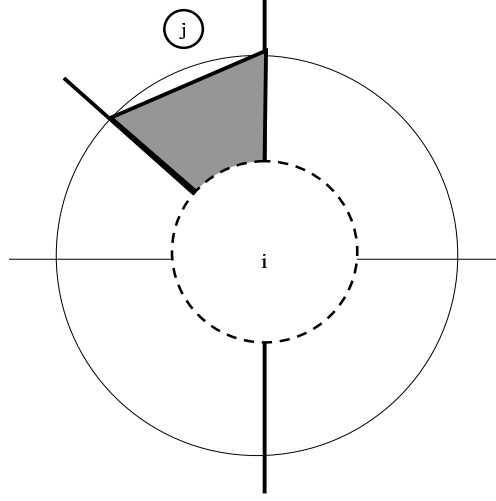


Fig. 1. Current relay node i and its relative position to destination node at j . Angular radius of the larger circle is θ_k where as the radius of the dotted circle is $\theta_k/2$.

$\mu = NS/A = 4 \log N$. By Chernoff bound, the probability that the actual number of nodes is not in the interval $(0.5\mu, 1.5\mu) = (2 \log N, 6 \log N)$ is at most $e^{-(1/2)^3 \times 16 \log N} < N^{-2}$. ■

The following is a simple trigonometric inequality, the proof of which is omitted.

Lemma 4.2: $\forall \theta \in (0, \pi], 1/4 < \frac{1 - \cos(\theta/2)}{1 - \cos(\theta)} < 1/2$.

Next,

Lemma 4.3: Assume that a packet destined for a node m is currently at a node $i \neq m$. For any open channel $k \leq l(i)$ of i , either $m \in \Psi^k(i)$ in which case the packet will be directly forwarded to the target m through channel k or else w.h.p. there exists a node $s \in \Psi_k$ that is $\epsilon_k/2$ closer to m than i , that is: $\angle(s, m) \leq \angle(i, m) - \epsilon_k/2$.

Proof: If $m \in \Psi^k(i)$ the the packet will of course be sent to m directly. Otherwise, consider two caps of angles $\theta_k, \theta_k/2$ centered at the current relay node i . Without loss of generality, assume the target is in the first octant around the node i as in Fig. 1. Therefore any node in area indicated by B is closer to m than the node i . The area of B is $2\pi R(1 - \cos(\theta_k/2)) \geq (1/4)2\pi R(1 - \cos(\theta_k)) = A_k/4 = 2^{k-2}A_0$, by using Lemma 4.2. By Lemma 4.1 on the other hand, this area contains at least $2 \log N$ nodes w.h.p., that have their k^{th} channel open. Each of these nodes is closer to m than i . Therefore, the packet will get an angle of at least $\epsilon_k = \theta_k/2$ closer to the target w.h.p. ■

Lemma 4.3 can now be used to prove the correctness of the routing in MRRH.

Theorem 4.4: The probability that there exists any unroutable pair of communication in the network is at most N^{-1} .

Proof: The proof follows from the fact that the base channel of all nodes are always open. Applying Lemma 4.3 to the base channel shows that the message gets an angle of least $\theta_0/2 \geq (64\pi \log N)/N$ degrees closer to the target at each hop, and thus it takes at most $\frac{N}{32\pi \log N}$ hops for any packet to be delivered. The probability of the failure is by union bound at most $N^{-2} \times N/(32\pi \log N) < N^{-1}$. ■

The correctness of MRRH is not enough for that to perform efficient data delivery. Average NL of MRRH is in fact only $O(\log^3 N)$, as is discussed next. The idea behind this result is the following: consider a packet starting from i and heading for a node j . If j is far from i , the packet will be communicated by means of links in upper channels (larger k) using longer range hops. As such, the packet gets quickly close to j . Once it gets to the vicinity of j , the routing will use shorter range connections.

The participation function of the MRRH, and the connectivity of MRRH topology at each level, ensure the success of the algorithm. The key ingredients of the proof are that if a packet is further from destination than the range of a channel k , and if the current node has a channel $k' > k$ open, the packet will always be routed through channel k' or a higher channel. Therefore, the packet *climbs up* the hierarchy, exploiting long range connections provided by nodes in the upper hierarchies to get close to its destination quickly. The structure of the MRRH ensures that finding nodes in upper hierarchies is always possible for all packets starting from any node in the network.

Let $C(i, j)$ denote the cap centered at i with angle $\angle(i, j)$. In other words, $C(i, j)$ is the cap centered at i that passes through j . Let $A(i, j) = A(j, i) = 2\pi R^2(1 - \cos(\angle(i, j)))$ denote the area of this cap.

We now move to the main Theorem of this section.

Theorem 4.5: The average NL of MRRH is $L_{avg} < 2 \log^3 N$.

Proof: Consider a packet heading towards a target node j . We say that routing is in phase g if the current position of the packet is node i and $\theta_g < \angle(j, i) \leq \theta_{g+1}$, or equivalently, when $A_g < A(i, j) \leq A_{g+1}$. The starting phase is at most $\log N$. When the packet reaches phase 0, it can be delivered to the target immediately through the base channel. We now show that after $2 \log N$ routing steps, the phase of the packet is decreased by at least 1 w.h.p.

To see this, assume a communication to be in phase $g > 1$. First assume that $l(i) \geq g + 1$, that is, i has its $g + 1^{th}$ channel open. Therefore, it can reach an area of size at least $A_k/4 = 2^{g-2}A_0$ of all nodes in phase $g - 1$ of the target (see Fig. 2). From lemma (4.1), this area contains at least $2 \log N$ nodes whose channel $g + 1$ are open w.h.p. Therefore, if $l(i) > g + 1$, w.h.p. the message will be passed to a node in phase $g - 1$ of the target.

If $l(i) < g + 1$, then by the routing algorithm, it passes the packet to the closest node to the target. From (4.3) we know that there is at least one node closer to the target than i w.h.p., hence the phase of the routing is never decreased.

There remains to show that for any $g > 0$, after at most $2 \log(N)$ steps, any packet can arrive at a node operating at a level greater than or equal to $g + 1$. The worst case is when the node in the starting position operates at channel 0 only, and the target is furthest ($\theta = 2\pi$) apart. It can be easily verified that by at most $2 \log(N)$ sampling of nodes working in channel g a node whose channel $g + 1$ is open can be found w.h.p., where as this takes two steps on average. If the packet is already in phase $g + 1$ we are finished. Since going to an upper channel takes less than $2 \log(N)$ steps and there are only K such channels, the total number of steps required to find a node in any channel is at most $2K \log N$ steps

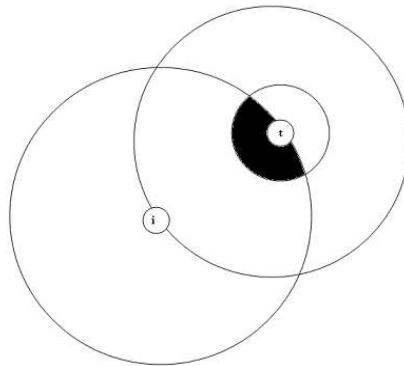


Fig. 2. The ideas in Theorem 4.5: When node i is in phase g of the target, the solid area is the one reached by all nodes in phase $g - 1$ of the target.

w.h.p. With these many steps, the phase of the routing is decreased by at least 1 w.h.p. The maximum phase to consider is $K < \log N$. Thus, any packet will reach the destination in at most $2K \log^2 N$ with high probability. Noting that $K < \log N$, we get the desired result. ■

Along the lines of proof of Theorem 4.5, we have also proved the following lemma that will prove useful in bounding the bandwidth requirement of MRRH nodes in later sections,

Lemma 4.6: If a message is being delivered to a target from channel k of a node in phase $g > k$ of the target, it almost surely leaves channel k for an upper channel after at most $2 \log(N)$ consecutive moves in channel k , independent of k .

So far, we have proved various routing properties of MRRH. In particular, we have shown that MRRH is routable, in the sense that any given packet from any node to any destination will be delivered in finite time w.h.p. More over, we have shown that the number of hops taken for the delivery of each packet is on average at most $2 \log^3 N$.

These observations are all geometric in nature. In the following sections, we derive the feasible throughput of MRRH for a given average power and total bandwidth.

V. POWER AND BANDWIDTH REQUIREMENT OF MRRH

In this section, we derive the power and bandwidth required by MRRH. To do so, we need to upper bound the rate of the communication required at any given channel. We then calculate the power and the amount of bandwidth required for communication in each channel. Finally, by comparison with results in propositions 3.2, 3.3, we show that the power and bandwidth requirements of MRRH are only polylogarithmically away from the lower bounds.

A. Bit Rate Requirement at Different Channels

We start by finding the rate with which each node i has to communicate with its neighbor nodes $\Psi^k(i)$ at each channel k in order to make a total throughput λ feasible. To do this, we need to upper bound

the probability that channel k of a random node i is used for a random routing, assuming that $l(i) \geq k$ (i.e., channel k of i is open). This will determine the amount of load that has to be handled by channel k of node i .

The next lemma shows that a packet does not *stay* in any channel k for a long time.

Lemma 5.1: Assume a packet, targeted for a node j , is routed to a node i through its k^{th} channel. If $j \notin \Psi^k(i)$, the next routing will happen in a channel $k' \geq k$ w.h.p.

Proof: This lemma states that the channel in which the packet is routed will decrease only when the packet gets close enough to the final target. The idea of the proof is that if the target is far, w.h.p. a node closer to it in channel k will be found and thus the level of the node receiving the packet does not decrease. The details of the proof are omitted. ■

Lemma 5.2: Let U^k be the number of times the k^{th} channel of a random node i is used for $N/2$ random packets. Then $U^k < 2^{k+2} \log N$ w.h.p.

Proof: Suppose a packet has started from a random node c towards a random destination j and is currently being relayed by a node i over its k^{th} channel. Two scenarios are possible: (1) If i is in phase $g \geq k + 1$ of the target, then through Lemma 5.1 the message will leave channel k after at most $2 \log N$ more steps in channel k w.h.p.. (2) If the message is in a phase $g < k + 1$, then through Lemma 5.1, it will immediately leave channel k w.h.p. Thus, the necessary condition for a channel k of any node i to be involved in the transmission of a packet is that either the target is in its neighborhood, i.e., $j \in \Psi^k(i)$, or by lemma 4.6, $\angle(i, c) < 2 \log(N) \theta_k$.

Let p_k be the probability that the k^{th} channel of a randomly chosen node i participates in a random communication from a randomly chosen source s to a destination t . By the union bound, p_k can be bounded as:

$$\begin{aligned} p_k &\leq Pr\{\angle(i, s) < 2\theta_k \log N\} + Pr\{t \in \Psi^k(i)\} \\ &\leq 4 \times 2^k \log N / N \end{aligned}$$

For $N/2$ communication pairs, the average number of times communication pairs for which channel k of node i has to relay packets is $2^{k+1} \log N$. Applying the Chernoff bound gives the result. ■

We can now use Lemma (5.2) to upper bound the average communication rate required for making a uniform throughput of λ feasible.

Theorem 5.3 (Bit-Rate): In MRRH, a communication rate of $R^k(i, j) = 24\lambda \log^2 N 2^k$ for any $k \leq K$ is sufficient for a uniform throughput of λ to be feasible. This can be achieved using a Time Division Multiple Access strategy.

Proof: From Lemma 5.2, for $N/2$ random source-destination communications, the maximum number of data packets that has to be communicated through a channel $k \leq K$ of any node i whose level $l(i)$ is greater than k is $2^{k+2} \log N$. For a throughput of λ bits/sec, this means that data with the rate of at most $\lambda 2^{k+2} \log N$ bits per second has to be communicated through channel k .

To resolve the conflicts we adopt a simple TDMA approach. We assume that when a node i needs to communicate in channel k , no other node in its neighborhood, $\Psi^k(i)$, will communicate in channel k . But from lemma 4.1, there are at most $6 \log N$ such nodes. This TDMA scheme can thus be used to yield communication turn to the nodes by slotting time into at most $6 \log_2 N$ slots in each channel. To communicate $\lambda 2^{k+2} \log N$ bits per second, the communication rate thus needs to be at most: $R^k(i, j) = 24\lambda 2^k \log N$ for any two nodes i, j communicating in channel k . ■

B. Power Requirement

Using Eqn. (1) we can find the power P^k necessary for providing the bit-rate required in (5.3). For now assume that the bandwidth at each channel is infinite, i.e., $B_k \rightarrow \infty$ for all $k \in \mathcal{K}$. Eqn. (1) will therefore reduce to a linear relation between the transmitted power P^k and the bit-rate R^k :

$$R^k(i, j) = \ln 2 P^k \gamma(\|X_i - X_j\|) / \eta_0 \quad (8)$$

Let's assume a specific power decay function of the form $\gamma(\|X_i - X_j\|) = \max\{(2\pi R)^{-d} (\angle(i, j))^{-d}, 1\}$. The “max” operation is necessary for γ to be a physical “loss” function.

Now note that two nodes will only communicate if $\angle(i, j) < \theta_k$. The bitrate demand in Lemma 5.3 can be satisfied by letting $P_k = \lambda \eta_0 \ln 2 (2\pi R)^d 2^k \theta_k^d \log^2 N$ when the bandwidths are infinite and thus the interferences can be neglected.

Now, the probability that a node i has its k^{th} channel open is at most 2^{-k} . As such, the average power consumption of a random node is bounded as:

$$P_{avg} \leq \sum_{k \in \mathcal{K}} 2^{-k} \times \lambda \eta_0 \ln 2 (2\pi R)^d 2^k \theta_k^d \log^2 N \quad (9)$$

$$\leq (\lambda \eta_0 \ln 2) (8\pi^2)^d (\log^3 N) R^d \quad (10)$$

C. Bandwidth Requirement

The average power requirement in (9) is found assuming an infinite bandwidth. We now upper bound the bandwidth requirement as a function of N as follows. First note that $\forall x > 0, \ln(1 + x) > x - x^2$. Using this, Eqn. (1) can be lower bounded as:

$$R_t^k(i, j) \geq \ln 2 P^k \gamma(\|X_i - X_j\|) (\kappa - \kappa^2) \quad (11)$$

where

$$\kappa \triangleq \frac{1}{1 + \frac{\sum_{l \in \Gamma, l \neq j} P^k \phi_t^k(j, l) \gamma(\|X_j - X_l\|)}{\eta_0 B_k}}$$

Therefore, we only need to choose B_k high enough to mask the interference. The interference can be bounded by noticing that in any given cap of angle θ_k there is at most one node transmitting in channel k at any given time. Now for a given node j , consider the sequence of caps of angle $\theta_k, 2\theta_k, 4\theta_k, \dots$

centered at j and call them C_1, C_2, C_3, \dots respectively. Now note that $(\text{area}(C_2) - \text{area}(C_1)) / \text{area}(C_1) = (1 - \cos(\theta_k)) / (1 - \cos(\theta_k/2)) - 1 \leq 3$, from Lemma 4.2. Therefore, at any given point of time, there can be at most 3 nodes that are in C_2 but not C_1 and are transmitting simultaneously. Likewise, for any m such that $2^m \theta_k < 2\pi$, it can be shown that there are at most 3×2^m nodes simultaneously transmitting on their k^{th} channel. The interference of these transmitting nodes on j can therefore be bounded as:

$$\begin{aligned} I^k &= \sum_{l \in \Gamma, l \neq j} P^k \phi_l^k(j, l) \gamma(\|X_j - X_l\|) \\ &\leq 6P_k \left(8\pi R \sqrt{2^k N^{-1} \log N} \right)^{-d} = 6P_k N^{d/2} (8\pi)^{-d} (2^k \log N)^{-d/2} \end{aligned}$$

eqn:rk

It therefore suffices to have $B_k > 6P_k N^{d/2} (8\pi)^{-d} (2^k \log N)^{-d/2}$, in which case, from (11):

$$R^k(i, j) \geq (\ln 2) P^k \gamma(\|X_i - X_j\|) / (4\eta_0)$$

Comparing with (8) this indicates a factor of at most 4 loss in the throughput.

The total bandwidth requirement is:

$$\begin{aligned} B &= \sum_{k=0}^K 6P_k N^{d/2} (8\pi)^{-d} (2^k \log N)^{-d/2} \\ &= 6\lambda\eta_0 \ln 2 (2\pi)^d \sum_{k=0}^K N^{d/2} (\log^{2-d/2} N) \tag{12} \\ &\leq 6\lambda\eta_0 \ln 2 (2\pi)^d N^{d/2} (\log^{3-d/2} N) \tag{13} \end{aligned}$$

We have then proved the following Theorem:

Theorem 5.4: MRRH can provide a constant throughput of λ with average NL of $O(\log^3 N)$ while requiring an average power of at most $P_{avg} = O(\lambda R^d \log^3 N)$ and a total bandwidth of at most $B = O(\lambda N^{d/2} \log^{3-d/2} N)$. For a constant node density ρ , and a direct line of sight path loss model ($d = 2$), MRRH requires $P_{avg} = O(\lambda \rho^{-1} N \log^3 N)$ and $B = O(\lambda \rho^{-1} N \log^2 N)$.

D. Near Optimality of MRRH

Comparing Theorem 5.4 with Theorem 3.2 and 3.3, the power and bandwidth consumption of MRRH are at most $O(\log^6 N)$ away from the absolutely most power and bandwidth aware communication systems.

VI. CONCLUDING REMARKS

Low NL wireless ad-hoc networking calls for significantly more power and bandwidth compared to nearest neighbor communication schemes. This is because, to achieve a low NL, many long range communications have to take place. Such communications require a significantly larger energy to perform. These long range communications will interfere with most other nodes in the network. Therefore, a mere

increase in the amount of the power used in communications is not enough; the bandwidth of the system should also increase to cancel the interferences caused by nodes communicating over long links.

In this paper, we derived a set of new lower bound to quantify the tradeoffs between power, bandwidth and NL. Results in propositions 3.3,3.2 put sever lower bounds on the power and bandwidth requirement of any low NL WANET. For a close to constant NL, one has to scale both bandwidth and average power at least linearly with N .

Our bounds are tightest for small NL. By methods close to the ones used in [12], one can show that even when the constraint on NL is relaxed, a constant throughput λ can be feasible only if $B = \Theta(\sqrt{N})$ and $P_{avg} = \Theta(\sqrt{N})$ and is achieved by nearest neighbor communication (NNC); note that nearest neighbor communication incurs an average NL of of $\Theta(\sqrt{N})$ hops. Therefore, to get a factor of \sqrt{N} reduction in NL compared to NNC, one should increase the average power consumption and bandwidth by at least a factor of \sqrt{N} .

Given the constraints on networks with low NL, we considered the question of designing efficient WANETs. We devised a system, called Multi-Resolution-Randomized-Hierarchy (MRRH) for efficient, low NL wireless ad-hoc networking. The efficiency of MRRH was proved by comparing its bandwidth and power requirements with our newly derived lower bounds.

MRRH is part of our ongoing research on implications of low NL communication on wireless ad-hoc and sensor networks [18].

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