Capacity constraints and the inevitability of mediators in adword auctions

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Abstract. One natural constraint in the sponsored search advertising framework arises from the fact that there is a limit on the number of available slots, especially for the popular keywords, and as a result, a significant pool of advertisers are left out. We study the emergence of diversification in the adword market triggered by such capacity constraints in the sense that new market mechanisms, as well as, new for-profit agents are likely to emerge to combat or to make profit from the opportunities created by shortages in ad-space inventory. We propose a model where the additional capacity is provided by for-profit agents (or, mediators), who compete for slots in the original auction, draw traffic, and run their own sub-auctions. The quality of the additional capacity provided by a mediator is measured by its *fitness* factor. We compute revenues and payoffs for all the different parties at a *symmetric Nash equilibrium* (SNE) when the mediator-based model is operated by a mechanism currently being used by Google and Yahoo!, and then compare these numbers with those obtained at a corresponding SNE for the same mechanism, but without any mediators involved in the auctions. Such calculations allow us to determine the value of the additional capacity. Our results show that the revenue of the auctioneer, as well as the social value (i.e. efficiency), always increase when mediators are involved; moreover even the payoffs of *all* the bidders will increase if the mediator has a high enough fitness. Thus, our analysis indicates that there are significant opportunities for diversification in the internet economy and we should expect it to continue to develop richer structure, with room for different types of agents and mechanisms to coexist.

1 Introduction

Sponsored search advertising is a significant growth market and is witnessing rapid growth and evolution. The analysis of the underlying models has so far primarily focused on the scenario, where advertisers/bidders interact directly with the auctioneers, i.e., the Search Engines and publishers. However, the market is already witnessing the spontaneous emergence of several categories of companies who are trying to mediate or facilitate the auction process. For example, a number of different AdNetworks have started proliferating, and so have companies who specialize in reselling ad inventories. Hence, there is a need for analyzing the impact of such incentive driven and for-profit agents, especially as they become more sophisticated in playing the game. In the present work, our focus is on the emergence of market mechanisms and for-profit agents motivated by capacity constraint inherent to the present models.

For instance, one natural constraint comes from the fact that there is a limit on the number of slots available for putting ads, especially for the popular keywords, and a significant pool of advertisers are left out due to this capacity constraint. We ask whether there are sustainable market constructs and mechanisms, where new players interact with the existing auction mechanisms to increase the overall capacity. In particular, lead-generation companies who bid for keywords, draw traffic from search pages and then redirect such traffic to service/product providers, have spontaneously emerged. However, the incentive and equilibria properties of paid-search auctions in the presence of such profit-driven players have not been explored. We investigate key questions, including what happens to the overall revenue of the auctioneers when such mediators participate, what is the payoff of a mediator and how does it dependent on her quality, how are the payoffs of the bidders affected, and is there an overall value that is generated by such mechanisms.

Formally, in the current models, there are K slots to be allocated among N ($\geq K$) bidders (i.e. the advertisers). A bidder i has a true valuation v_i (known only to the bidder i) for the specific keyword and she bids b_i. The expected *click through rate* (CTR) of an ad put by bidder i when allocated slot j has the form $\gamma_j e_i$ i.e. separable in to a position effect and an advertiser effect. γ_j's can be interpreted as the probability that an ad will be noticed when put in slot j and it is assumed that $\gamma_1 > \gamma_2 > \cdots > \gamma_K > \gamma_{K+1} = \gamma_{K+2} = \ldots \gamma_N = 0$. e_i can be interpreted as the probability that an ad put by bidder i will be clicked on if noticed and is refered as the *relevance* of bidder i. The payoff/utility of

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bidder i when given slot j at a price of p per click is given by $e_i\gamma_j(v_i - p)$ and they are assumed to be rational agents trying to maximize their payoffs. As of now, Google as well as Yahoo! uses schemes closely modeled as RBR(rank by revenue) with GSP(generalized second pricing). The bidders are ranked according to e_iv_i and the slots are allocated as per this ranks. For simplicity of notation, assume that the ith bidder is the one allocated slot i according to this ranking rule, then i is charged an amount equal to $\frac{e_{i+1}v_{i+1}}{e_i}$. Formal analysis of such sponsored search advertising model has been done extensively in recent years, from algorithmic as well as from game theoretic perspective[2, 6, 3, 1, 7, 4, 5].

In the following section, we propose and study a model wherein the additional capacity is provided by a for-profit agent who competes for a slot in the original auction, draws traffic and runs its own sub-auction for the added slots. We discuss the cost or the value of capacity by analyzing the change in the revenues due to added capacity as compared to the ones without added capacity.

2 The Model

In this section, we discuss our model motivated by the capacity constraint, which can be formally described as follows:

– **Primary Auction (p-auction) :** Mediators participate in the original auction run by the search engine (called p-*auction*) and compete with advertisers for slots (called *primary slots*). For the ith agent (an advertiser or a mediator), let v_i^p and b_i^p denote her true valuation and the bid for the p-auction respectively. Further, let us denote $v_i^p e_i^p$ by s_i^p where e_i^p is the relevance score of ith agent for p-auction. Let there are κ mediators and there indices are $M_1, M_2, \ldots, M_\kappa$ respectively.

– **Secondary auctions (s-auctions):**

- **Secondary slots:** Suppose that in the primary auction, the slots assigned to the mediators are $l_1, l_2, \ldots, l_\kappa$ respectively, then effectively, the additional slots are obtained by forking these *primary slots* in to $L_1, L_2, \ldots, L_\kappa$ additional slots respectively, where $L_i \leq K$ for all $i = 1, 2, \ldots, \kappa$. By forking we mean the following: on the associated landing page the mediator puts some information relevant to the specific keyword associated with the p-auction along with the space for additional slots. Let us call these additional slots as *secondary slots*.

- **Properties of secondary slots and *fitness* of the mediators:** For the ith mediator, there will be a probability associated with her ad to be clicked if noticed, which is actually her relevence score $e^p_{M_i}$ and the position based CTRs might actually improve say by a factor of α_i. This means that the position based CTR for the jth secondary slot of ith mediator in modeled as $\alpha_i\gamma_j$ for $1 \leq j \leq L_i$ and 0 otherwise. Therefore, we can define a *fitness* f_i for the ith mediator, which is equal to $e^p_{M_i}\alpha_i$. Thus corresponding to the l_ith primary slot (the one being forked by the ith mediator), the *effective* position based CTR for the jth secondary slot obtained is $\tilde{\gamma}_{i,j}$ where

$$\tilde{\gamma}_{i,j} = \begin{cases} \gamma_{l_i} f_i \gamma_j & \text{for } j = 1, 2, \ldots, L_i, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

  Note that $f_i\gamma_1 < 1$, however f_i could be greater than 1.

- **s-auctions:** Mediators run their individual sub-auctions (called s-*auctions*) for the secondary slots provided by them. For an advertiser there is another type of valuations and bids, the ones associated with s-auctions. For the ith agent, let $v^s_{i,j}$ and $b^s_{i,j}$ denote her true valuation and the bid for the s-auction of jth mediator respectively. In general, the two types of valuations or bids corresponding to p-auction and the s-auctions might differ a lot. We also assume that $v^s_{i,j} = 0$ and $b^s_{i,j} = 0$ whenever i is a mediator. Further, for the advertisers who do not participate in one auction (p-auction or s-auction), the corresponding true valuation and the bid are assumed to be zero. Also, for notational convenience let us denote $v^s_{i,j}e^s_{i,j}$ by $s^s_{i,j}$ where $e^s_{i,j}$ is the relevance score of ith agent for the s-auction of jth mediator.

- **Payment models for s-auctions:** Mediators could sell their secondary slots by impression (PPM), by pay-per-click (PPC) or pay-per-conversion(PPA). In the following analysis, we consider PPC.

– **Freedom of participation:** Advertisers are free to bid for primary as well as secondary slots.

– **True valuations of the mediators:** The true valuation of the mediators are derived from the expected revenue (total payments from advertisers) they obtain from the corresponding s-auctions[1] *ex ante*.

3 Bid Profiles at SNE

For simplicity, let us assume participation of a single mediator and the analysis involving several mediators can be done in a similar fashion. For notational convenience let

$$
\begin{aligned}
f = f_1, &\text{ the fitness of the mediator} \\
l = l_1, &\text{ the position of the primary slot assigned to the mediator} \\
L = L_1, &\text{ the number of secondary slots provided by the mediator in her } s\text{-auction} \\
M = M_1, &\text{ the index of the mediator i.e. } M\text{th agent is the mediator} \\
\tilde{\gamma}_j = \tilde{\gamma}_{1,j}, &\text{ is the \textit{effective} position based CTR of the } j\text{th secondary slot provided by the mediator} \\
v^s_{i,1} = v^s_i, &\text{ is the true valuation of the agent } i \text{ for the } s\text{-auction} \\
b^s_{i,1} = b^s_i, &\text{ is the bid of the agent } i \text{ for the } s\text{-auction, and} \\
s^s_{i,1} = s^s_i = v^s_i e^s_i, &\text{ where } e^s_i = e^s_{i,1} \text{ is the relevance score of } i\text{th agent for the } s\text{-auction.}
\end{aligned}
$$

The p-auction as well as the s-auction is done via *RBR* with *GSP*, i.e. the mechanism currently being used by Google and Yahoo!, and the solution concept we use is *Symmetric Nash Equilibria(SNE)*[2, 7]. Suppose the allocations for the p-auction and s-auction are $\sigma : \{1, 2, \ldots, N\} \longrightarrow \{1, 2, \ldots, N\}$ and $\tau : \{1, 2, \ldots, N\} \longrightarrow \{1, 2, \ldots, N\}$ respectively. Then the payoff of the ith agent from the combined auction (p-auction and s-auction together) is

$$u_i = \gamma_{\sigma^{-1}(i)} \left(s^p_i - r^p_{\sigma^{-1}(i)+1} \right) + \tilde{\gamma}_{\tau^{-1}(i)} \left(s^s_i - r^s_{\tau^{-1}(i)+1} \right)$$

where

$$
\begin{aligned}
r^p_j &= b^p_{\sigma(j)} e^p_{\sigma(j)}, \\
r^s_j &= b^s_{\tau(j)} e^s_{\tau(j)}.
\end{aligned}
$$

From the mathematical structure of payoffs and strategies available to the bidders wherein two different uncorrelated values can be reported as bids in the two types of auctions independently of each other[2], it is clear that the equilibrium of the combined auction game is the one obtained from the equilibria of the p-auction game and the s-auction game each played in isolation. In particular at *SNE*[2, 7],

$$\gamma_i r^p_{i+1} = \sum_{j=i}^{K} (\gamma_j - \gamma_{j+1}) s^p_{\sigma(j+1)} \text{ for all } i = 1, 2, \ldots, K$$

and

$$\tilde{\gamma}_i r^s_{i+1} = \sum_{j=i}^{L} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) s^s_{\tau(j+1)} \text{ for all } i = 1, 2, \ldots, L$$

which implies that (see Eq. (1))

$$\gamma_i r^s_{i+1} = \sum_{j=i}^{L-1} (\gamma_j - \gamma_{j+1}) s^s_{\tau(j+1)} + \gamma_L s^s_{\tau(L+1)} \text{ for all } i = 1, 2, \ldots, L$$

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[1] This way of deriving the true valuation for the mediator is reasonable for the mediator can participate in the p-auction several times and run her corresponding s-auction and can estimate the revenue she is deriving from the s-auction.

[2] This assumption was motivated by some empirical examples from Google Adword[3].

where

$$s^p_{\sigma(l)} = s^p_M = f\sum_{j=1}^{L}\gamma_j r^s_{j+1} = f\left(\sum_{j=1}^{L-1}(\gamma_j-\gamma_{j+1})js^s_{\tau(j+1)} + \gamma_L L s^s_{\tau(L+1)}\right)$$

is the true valuation of the mediator multiplied by her relevance score as per our definition[1], which is the expected revenue she derives from her s-auction *ex ante* given a slot in the p-auction and therefore the mediator's payoff at SNE is

$$u_M = \gamma_l f\left(\sum_{j=1}^{L-1}(\gamma_j-\gamma_{j+1})js^s_{\tau(j+1)} + \gamma_L L s^s_{\tau(L+1)}\right) - \sum_{j=l}^{K}(\gamma_j-\gamma_{j+1})s^p_{\sigma(j+1)}.$$

4 Revenue of the Auctioneer

In this section, we discuss the change in the revenue of the auctioneer due to the involvement of the mediator. The revenue of the auctioneer with the participation of the mediator is

$$R = \sum_{j=1}^{K}\gamma_j r^p_{j+1} = \sum_{j=1}^{K}(\gamma_j-\gamma_{j+1})js^p_{\sigma(j+1)}$$

and similarly, the revenue of the auctioneer without the participation of the mediator is

$$R_0 = \textstyle\sum_{j=1}^{K}(\gamma_j-\gamma_{j+1})js^p_{\tilde{\sigma}(j+1)} \text{ where } \tilde{\sigma}(j)=\sigma(j) \text{ for } j<l \text{ and } \tilde{\sigma}(j)=\sigma(j+1) \text{ for } j\geq l$$

$$= \textstyle\sum_{j=1}^{l-2}(\gamma_j-\gamma_{j+1})js^p_{\sigma(j+1)} + \sum_{j=l-1}^{K}(\gamma_j-\gamma_{j+1})js^p_{\sigma(j+2)}.$$

Therefore,

$$R - R_0 = \sum_{j=max\{1,l-1\}}^{K}(\gamma_j-\gamma_{j+1})j(s^p_{\sigma(j+1)} - s^p_{\sigma(j+2)})$$

$$\geq 0 \text{ as } s^p_{\sigma(i)} \geq s^p_{\sigma(i+1)} \forall i = 1,2,\ldots,K+1 \text{ at } SNE.$$

Thus revenue of the auctioneer always increases by the involvement of the mediator. As we can note from the above expression, smaller the l better the improvement in the revenue of the auctioneer. To ensure a smaller value of l, the mediator's valuation which is the expected payments that she obtains from the s-auction should be better, therefore fitness factor f should be very good. There is another way to improve her true valuation. The mediator could actually run many subauctions related to the specific keyword in question. This can be done as follows: besides providing the additional slots on the landing page, the information section of the page could contain links to other pages wherein further additional slots associated with a related keyword could be provided[3]. With this variation of the model, a better value of l could possibly be ensured leading to a win-win situation for everyone.

Theorem 1 *Increasing the capacity via mediator improves the revenue of auctioneer.*

5 Efficiency

Now let us turn our attention to the change in the efficiency and as we will prove below, the efficiency always improves by the participation of the mediator.

[3] For example, the keyword "personal loans" or "easy loans" and the mediator "personalloans.com".

$$E_0 = \sum_{j=1}^{K} \gamma_j s^p_{\tilde{\sigma}(j)} = \sum_{j=1}^{l-1} \gamma_j s^p_{\sigma(j)} + \sum_{j=l}^{K} \gamma_j s^p_{\sigma(j+1)} \text{ and}$$

$$E = \sum_{j=1}^{l-1} \gamma_j s^p_{\sigma(j)} + \sum_{j=l+1}^{K} \gamma_j s^p_{\sigma(j)} + \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)}$$

$$\begin{aligned}
\therefore E - E_0 &= \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} - \sum_{l}^{K} (\gamma_j - \gamma_{j+1}) s^p_{\sigma(j+1)} \\
&= \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} - \gamma_l r^p_{l+1} \\
&\geq 0 \\
&\text{as } \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} \geq \gamma_l f \sum_{j=1}^{L} \gamma_j r^s_{j+1} = \gamma_l s^p_{\sigma(l)} \geq \gamma_l r^p_{l+1} \text{ at } \textit{SNE} .
\end{aligned}$$

Theorem 2 *Increasing the capacity via mediator improves the efficiency.*

6 Advertisers' Payoffs

Clearly, for the newly accommodated advertisers, that is the ones who lost in the p-auction but win a slot in s-auction, the payoffs increase from zero to a postitive number. Now let us see where do these improvements in the revenue of the auctioneer, in payoffs of newly accommodated advertisers, and in the efficiency come from? Only thing left to look at is the change in the payoffs for the advertisers who originally won in the p-auction, that is the winners when there was no mediator. The new payoff for jth ranked advertiser in p-auction is

$$u_{\sigma(j)} = \gamma_j s^p_{\sigma(j)} - \sum_{i=j}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+1)} + u^s_{\sigma(j)}$$

where

$$u^s_{\sigma(j)} = \gamma_l f \gamma_{\tau^{-1}(\sigma(j))} \left(s^s_{\sigma(j)} - r^s_{\tau^{-1}(\sigma(j))+1} \right)$$

is her payoff from the s-auction. Also, for $j \leq l-1$, her payoff when there was no mediator is

$$\begin{aligned}
u^0_{\sigma(j)} &= \gamma_j s^p_{\sigma(j)} - \textstyle\sum_{i=j}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\tilde{\sigma}(i+1)} \\
&= \gamma_j s^p_{\sigma(j)} - \textstyle\sum_{i=j}^{l-2} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+1)} - \sum_{i=l-1}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+2)}. \\
\therefore u_{\sigma(j)} - u^0_{\sigma(j)} &= u^s_{\sigma(j)} - \textstyle\sum_{i=l-1}^{K} (\gamma_i - \gamma_{i+1}) (s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})
\end{aligned}$$

Similarly, for $j \geq l+1$, her payoff when there was no mediator is

$$\begin{aligned}
u^0_{\sigma(j)} &= \gamma_{j-1} s^p_{\sigma(j)} - \textstyle\sum_{i=j-1}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+2)} \\
\therefore u_{\sigma(j)} - u^0_{\sigma(j)} &= u^s_{\sigma(j)} - \textstyle\sum_{i=j-1}^{K} (\gamma_i - \gamma_{i+1}) (s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})
\end{aligned}$$

Therefore, in general we have,

$$u_{\sigma(j)} - u^0_{\sigma(j)} = u^s_{\sigma(j)} - \sum_{i=max\{l-1,j-1\}}^{K} (\gamma_i - \gamma_{i+1}) (s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)}).$$

Thus, for the jth ranked winning advertiser from the auction without mediation, the revenue from the p-auction decreases by $\sum_{i=max\{l-1,j-1\}}^{K}(\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})$ and she faces a loss unless compensated for by her payoffs in s-auction. Further, this payoff loss will be visible only to the advertisers who joined the auction game before the mediator and they are likely to participate in the s-auction so as to make up for this loss. Thus, via the mediator, a part of the payoffs of the originally winning advertisers essentially gets distributed among the newly accommodated advertisers. However, when the mediator's fitness factor f is very good, it might be a win-win situation for everyone. Depending on how good the fitness factor f is, sometimes the payoff from the s-auction might be enough to compensate for any loss by accommodating new advertisers. Let us consider an extreme situation when $L = K$ and $\tau = \tilde{\sigma}$. The *gain* in payoff for the advertiser $\sigma(j)$ is

$$\gamma_l f \sum_{i=j}^{K}(\gamma_i - \gamma_{i+1})(s^s_{\sigma(j)} - s^s_{\sigma(i+1)}) - \sum_{i=max\{l-1,j-1\}}^{K}(\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})$$

Therefore as long as

$$f \geq \frac{\sum_{i=max\{l-1,j-1\}}^{K}(\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})}{\gamma_l \sum_{i=j}^{K}(\gamma_i - \gamma_{i+1})(s^s_{\sigma(j)} - s^s_{\sigma(i+1)})}$$

the advertiser $\sigma(j)$ faces no net loss in payoff and might actually gain.

7 Concluding Remarks

In the present work, we have studied the emergence of diversification in the adword market triggered by the inherent capacity constraint. We proposed and analyzed a model where additional capacity is created by a for-profit agent who compete for a slot in the original auction, draws traffic and runs its own sub-auction. Our study potentially indicate a 3-fold diversification in the adword market in terms of (i) the emergence of new market mechanisms, (ii) emergence of new for-profit agents, and (iii) involvement of a wider pool of advertisers. Therefore, we should expect the internet economy to continue to develop richer structure, with room for different types of agents and mechanisms to coexist. In particular, capacity constraints motivates the study of yet another model where the additional capacity is created by the search engine itself, essentially acting as a mediator itself and running a single combined auction. This study will be presented in an extended version of the present work.

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