Distributed Resource Sharing in Low-Latency Wireless Ad Hoc Networks

Behnam A. Rezaei, Nima Sarshar, and Vwani P. Roychowdhury

***Abstract*—With the growing abundance of portable wireless communication devices, a challenging question that arises is whether one can efficiently harness the collective communication and computation power of these devices. In this paper, we investigate this question by studying a streaming application. Consider a network of $\mathcal{N}$ wireless nodes, each of power P, in which one or more nodes are interested in receiving a data stream from a fixed server node S. We ask whether distributed communication mechanisms exist to route media packets from S to the arbitrary but fixed receiver, such that 1) the average communication delay L is short, 2) the load is balanced, i.e., *all* nodes in the ensemble spend roughly the same amount of average power, and, more importantly, 3) power resources of all nodes are optimally shared, i.e., the lifetime of the network is comparable to an optimally designed network with L nodes whose total power is $\mathcal{N} \times P$.**

We develop a theoretical framework for incorporation of random long range routes into wireless ad hoc networking protocols that can achieve such performance. Surprisingly, we show that wireless ad hoc routing algorithms, based on this framework, exist that can deliver this performance. The proposed solution is a randomized network structuring and packet routing framework whose communication latency is only $L = O(\log^2 \mathcal{N})$ hops, on average, compared to $O(\sqrt{\mathcal{N}})$ in nearest neighbor communications while distributing the power requirement almost equally over all nodes. Interestingly, all network formation and routing algorithms are completely *decentralized,* and the packets arriving at a node are routed randomly and independently, based only on the source and destination locations. The distributed nature of the algorithm allows it to be implemented within standard wireless ad hoc communication protocols and makes the proposed framework a compelling candidate for harnessing collective network resources in a *truly large-scale* wireless ad hoc networking environment.

***Index Terms*—Low latency, multipath routing, resource sharing, scalability, small world, wireless ad hoc networks.**

I. Introduction

PORTABLE wireless communication devices have become central parts of modern life. In a given day, thousands of wireless-enabled devices might be present in a busy urban area. The activity pattern of these devices is often variable; at any given time, most devices are in idle mode, while a small percentage of them are actively seeking and receiving data streams. A tempting question is whether one can tap into this pool of underutilized computing and communication resources to provide services that are otherwise extremely expensive or resource intensive to provide. We investigate this possibility for a typical wireless streaming application, in which the goal is to deliver packets of a media stream from a fixed but arbitrary source node in the network to one or more receivers, or clients. The source node may represent a base station with a wired connection to the Internet, and the clients can be nodes interested in receiving a live TV program, for instance. This problem is becoming increasingly more relevant with the recent interest in wireless and mobile media delivery services. In this paper, we investigate the following challenging problem from a rather theoretical ground; do wireless ad hoc routing algorithms exist to utilize the power and bandwidth resources of *all* network nodes to facilitate the streaming task at hand?

Our wireless ad-hoc networking (WANET) solution should share the following general properties with most other WANETs. First, the network infrastructure and *node connections* should be managed and maintained through a decentralized protocol. Secondly, the resulting network must be routable. Since the set of receivers is not known a priori, the network should be able to route packets from the source node to any other node in the network. Lastly, the algorithms used for routing the packets should also be *decentralized*. Ideally, the intermediate nodes should be able to route a packet based only on the information about the set of client nodes.

We also seek more specific properties from our WANET. Let us start with the issue of network latency. We call *the number of hops* a WANET uses for an average communication its *network latency* (NL). The name is chosen because this form of latency is induced by the network layer and is a reflection of the underlying routing algorithm.[1] Note that NL is purely geometric. Under low congestion assumption, the physical packet delay will be linearly proportional to NL. We require the NL of our WANET to be low. Ideally, we want the NL to scale logarithmically with the network size. A large NL could significantly impact the system performance of any WANET as it increases the communication delay due to buffering at relay nodes, which may require excessive number of packet retransmissions, which in turn would increase average power consumption (see, e.g., [13]–[16]).

A small NL, however, is power extensive. The overall power required for packet delivery increases superlinearly with the de-

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B. A. Rezaei and V. P. Roychowdhury are with the Department of Electrical Engineering, University of California, Los Angeles, CA 90029 USA.

N. Sarshar is with the Faculty of Engineering, University of Regina, Regina, SK S4S 0A2, Canada (e-mail: nima.sarshar@uregina.ca).

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[1]Note that network latency, unlike delay, is independent of the communication rate. Delay, on the other hand, is a measure of the amount of time it takes for a data packet to travel between two points. As pointed out in this paper, the average delay is usually linearly proportional to the network latency (due to buffering at relays) and inversely proportional to communication rate.

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TABLE I

SUMMARY OF THE SCALING PROPERTIES OF DIFFERENT ROUTING METHODS ON PROPOSED ABSTRACT MODEL WITH N (SIZE OF THE GRID) FOR $\alpha \geq 2$. *NOTE* THAT THE TOTAL ENERGY REQUIRED FOR A PACKET TO BE TRANSMITTED, AND AVERAGE ENERGY CONSUMED PER NODE, FOR THE PROPOSED METHOD ARE AT MOST POLYLOGARITHMICALLY (IN THE SIZE PARAMETER) LARGER THAN THE MINIMUM REQUIRED. HENCE, THE PROPOSED METHOD UTILIZES THE BATTERY RESOURCES OF THE ENTIRE SET OF NODES, IN AN *ALMOST* OPTIMAL FASHION WHILE WORKING IN A DECENTRALIZED FASHION

↓Metric Method ⟶	Nearest Neighbor	Direct	Proposed	[4]
Latency	$O(N)$	O(1)	$O(\log^2 N)$	$O(N)$
Maximum Average Power Consumption at any node (per packet)	$O(P_0)$	$O(P_0 N^\alpha)$	$O(P_0 N^{\alpha-2} \log^2 N)$	$O(P_0)$
Total Energy consumed Per Packet	$O(P_0 N)$	$O(P_0 N^\alpha)$	$O(P_0 N^\alpha)$	$O(P_0 N)$
Lower Bound on Total Energy required per packet	$\Omega(P_0 N)$	$\Omega(P_0 N^\alpha)$	$\Omega(P_0 N^\alpha \log^{-2\alpha} N)$	$\Omega(P_0 N)$

crease in network latency (see Section II and Table I). As such, the rich and extensive literature on routing in WANETs has mainly been concerned with minimizing the overall power consumption, with little regard to its influence on network latency. As such, most routing algorithms focus on local communication, where packets are passed to nodes in close spatial proximity to reduce the power requirement at the expense of large NL (see, e.g., [1] and [9]–[12]).

Another important challenge that we face is the highly nonuniform traffic pattern in our application of interest. At any time, only a small subset of nodes are interested in receiving the streaming packets, and the rest of the nodes are acting as relays. We seek load balancing for *any* such demand pattern. As such, we will not be able to rely on the uniformity in the traffic to provide us with a "natural" load balancing. Simple local communication strategies, for instance, might result in overloading a few nodes close to the line of sight communication path while not utilizing the battery resources of other nodes. The use of deterministic *multipath collaborative routing* might spread the load on a larger fraction of nodes in the network. But the question that remains is whether we are able to use the battery resources of this larger set of nodes effectively. We can quantify this efficiency by answering the following question: given a certain battery life for each wireless node in the network, how many data packets can we communicate using our scheme compared to, say, an optimal strategy, where the power *of all the nodes* is concentrated and used as necessary to route packets with the same latency? Lastly, we need to quantify the cost incurred by our load spreading and resource sharing strategy. In particular, how much more power is spent to communicate a single packet, compared to a power-optimal communication strategy? (See Table I for the related metrics.)

Perhaps the most important contribution of this paper is to show that for the highly nonuniform application at hand, low-latency communication is feasible with scalable power overhead *on all nodes*. We accomplish this by distributing the power consumption almost uniformly over all nodes in the network, effectively *pooling* their battery resources, despite the highly nonuniform traffic pattern. The key is to occasionally route packets to spatially faraway nodes, according to a carefully chosen probabilistic rule. Our routing algorithm uses a *randomized* multipath collaborative routing to evenly distribute the load on all nodes in the network, which resembles the multipath routing algorithm of Servetto *et al.* [4]. The work of Servetto *et al.* [4], however, still relies on short-range communication and incurs a substantial packet latency. For example, in our system, we show that the average power consumption per node remains *almost constant* (in the case of free-space communication) and does not increase with the increase in the network size. In other words, even though the average communication length increases as the network size grows, the average power consumption per node remains constant, since the new nodes are able to effectively share their battery resources with others.

A. Organization of This Paper

The rest of this paper is organized as follows. In Section II, we formally introduce our grid based model of a wireless network and introduce the figures of merit with which we will analyze the performance of routing protocols on large-scale networks. We also delineate a fundamental tradeoff between two important performance metrics: the per-node power consumption and the latency. This section also contains a statement of the results derived in this paper. In Section III, we describe, in detail, the decentralized random-walk algorithms adopted in this paper for an idealized grid network topology. *While these algorithms are simple to implement, their analysis can be challenging*. The related results are derived in Section IV and in the Appendix. Extensive simulation results that verify our predictions are presented in Section V. While our developments in this paper are rather theoretical, the distributed nature of our algorithms allows for their implementation on top of de facto standard wireless ad hoc protocols, as discussed in Section VI.

II. PROBLEM MODEL AND STATEMENT OF RESULTS

1) Topology: We consider an $N+1$ by $N+1$ ($N \gg 1$) grid of nodes $\{(i,j)| -N/2 < i < N/2, -N/2 < j < N/2\}$, where the distance between adjacent nodes in the grid is one unit length. We note that models based on random placement of nodes in two dimensions with a constant density could have been adopted, resulting in essentially the same conclusions as this paper. Using a regular deterministic grid model, however, simplifies analytical derivations in later sections. The assumption of $N \gg 1$ will allow us to approximate summations over grid statistics with their Riemann integrals.

2) Path-Loss and Packet Communication Model: We assume a path-loss model of $r^{-\alpha}$, where r is the Euclidean distance and $\alpha \geq 2$ depends on the physical model. The communication model is as follows: a packet transmission is possible if and only if the signal-to-noise ratio (SNR) at the receiver node exceeds a constant $snr_{\min}$. For normalization, we assume P_0 is the transmission power required at any node to provide an SNR of $snr_{\min}$ at each of its four nearest neighbors, assuming the interference from all other nodes can be neglected. We assume that the transmission of each packet happens in unit time; thus, energy and power consumption are numerically equal, and hence are used interchangeably. An ultra-wide-band communication model is assumed in which transmissions between pairs of nodes do not interfere with each other. Such models are gaining popularity in ad hoc wireless networking problems [6]. With this assumption,[2] and when congestion can be neglected in relay nodes, the communication latency will only depend on the number of hops.

3) Routing Problem: The routing problem is defined as follows. A node S called the source node transmits a stream of data packets. M randomly selected nodes in the network are interested in receiving these data packets. The set of receiver nodes is randomly selected at the start of the communication but remains fixed throughput.

4) Location Based Routing: We are assuming in this model, just as in the work in [4], that nodes have access to positioning information. To analyze a fixed transmission pair, we assume, without loss of generality, that the source node is at the origin. While the model looks asymmetrical, with the source as the topological center, it is meant to highlight the communication from that particular source. One way to make the topology symmetric, regardless of the choice of source node, is to assume a grid on the surface of a sphere (see [7]). This again will only complicate the derivations without making any significant difference in the conclusions.

A. Results on Latency and Power Efficiency

There is a simple *tradeoff* between average power requirement and latency. For the network model described above, the total power requirement for a routing with latency Δ between two nodes that are a distance $\Theta(N)$ apart is easily shown to be at least $\Omega(N^{\alpha}\Delta^{-\alpha}P_0)$. This is because, for a routing with latency Δ, there is at least one communication with range $\Omega(N/\Delta)$, which is $\Omega(N/\Delta)$ times the range of nearest neighbor communication and requires a factor of $\Omega((N/\Delta)^{\alpha})$ more energy. Thus, if we want the latency to scale as $O(\log^{\beta} N)$, for instance, the total energy consumed per packet must increase at least as $\Omega(N^{\alpha}\log N^{-\beta\alpha})$.

Now take any single-path deterministic routing algorithm; all packets for a fixed source–destination communication have to follow a fixed path. This will result in overloading of at least one of the nodes in the path if the total latency is to be kept small. In fact, from the above arguments, there should be at least one node that consumes $\Theta(N^{\alpha}\Delta^{-\alpha}P_0)$ energy per packet. Single path, nearest neighbor, and direct communication are special cases of deterministic routing. For nearest neighbor communication, the average latency is $\Delta_{\text{near}} = \Theta(N)$. Each node on the path will consume a power P_0. As such, maintaining a communication stream will on average require a total power of $P = \Theta(NP_0)$. This power is evenly distributed on all nodes in the path. Thus the average power consumption of nodes in the path is $\Theta(P_0)$. Direct communication, on the other hand, will require $\Theta(N^{\alpha}P_0)$ energy per packet, and all this energy has to be provided by the source, resulting in serious power problems at either of these nodes.

[2]In Section A of the Appendix, we show that even in the presence of interference, the bandwidth requirement will increase only polylogarithmically in the size of the grid. Thus, the requirement of an ultra-wide-band communication model can be relaxed.

We now ask whether it is possible, perhaps by randomized routing, to achieve scalable latency (like the case of direct communication) while keeping the *per-node* power requirement scalable (like the case of nearest neighbor communication). In other words, while requiring low latency necessitates a large overall power consumption, can we distribute this power consumption evenly over all nodes, regardless of the position of the source and destination? Since there are $O(N^2)$ nodes in the network, the question is whether there is a scheme with average power per node (for each packet) of $O(N^{\alpha-2}\log^{-\beta\alpha} N)$ and latency of $O(\log^{\beta} N)$. This paper shows how one can almost reach this bound in a decentralized manner.

B. Results on Load Balance

Consider packet communication to or from a node O using nearest neighbor communications. Now consider a circle of radius r around O and let us communicate K packets using nearest neighbor communication. The number of nodes "close" to the perimeter of this circle scales linearly with r (since $2\pi r$ is the perimeter of this circle). Thus, there exists a node along the perimeter that has to communicate at least $\sim K/r$ of these packets. In other words, the maximum traffic on nodes at distance r from the origin diverges as r^{-1}, when nearest neighbor communication is used.

In our system, however, as the results derived in Section IV show, the power requirement (per packet) for a node at distance r from a source or a destination node is only $O\left(P_0 r^{\alpha-2}\log r \log N\right)$; for free-space communication, i.e., $\alpha = 2$, the power requirement per node is almost constant throughout the network (depends at most logarithmically on r). Table I summarizes the results and comparisons.

The main idea behind these results is the following. Nodes that are closer to the source or destination will be more likely to receive a packet for relaying. Once they receive a packet, however, they usually relay it to a node that is not very far away from them. On the other hand, nodes that are far from the source and destination will receive packets less often. Once they get the packet, however, they will usually have to use long-range connections to "throw" the packet either away from the source or towards the destination. The combination of these two will result in an almost constant power consumption over all the nodes in the network regardless of their position relative to source or destination.

III. Proposed WANET Solution

In this section, we propose random, decentralized routing algorithms that use collaboration of all the nodes in the network, instead of relying on a finite subset of the nodes. A new routing path is generated from source to destination for each batch of transmissions. Moreover, local and independent decisions at the node level are used to construct each new routing path.

The large-scale network model on which our routing algorithm operates is inspired by the small-world model studied by Kleinberg [5]. The proposed algorithm relies on two local packet routing subalgorithms: the contraction random walk algorithm (CRWA) and expansion random walk algorithm (ERWA). The CRWA is used to guide packets towards a given destination, while the ERWA is used to move packets out of the source node. The successive application of ERWA followed by CRWA forms our overall routing strategy.

A. Contraction Random Walk Algorithm

The CRWA is a way for each node to decide on where to route a received packet, which is destined for a certain receiver.

In CRWA, a path is established by passing a token along a random walk as specified below.

1) Only the position of the destination node (referred to as the node t) is required in routing. This information has to be appended to all data packets.
2) *Choosing a shortcut link:* Any node v selects a long-range contact node u from the set of all nodes. This link will be referred to as a *shortcut* link. The probability of a node u's being selected by v as a long-range contact is $\frac{1/d(u,v)^2}{\Sigma_{\forall u} 1/d(u,v)^2}$, where $d(u,v)$ is defined as the Euclidean distance between u and v, i.e., $d((i,j),(k,l)) = \sqrt{(k-i)^2+(l-j)^2}$.
3) *Deciding between a shortcut versus a local link:* If $d(v,t) > d(u,t)$ (where t is the destination node), then v passes the token to u. If $d(v,t) \leq d(u,t)$, then v passes the token to its nearest neighbor closer to t.
4) Step 3) is repeated until the token reaches the destination node t.
5) *Refreshing shortcuts:* Each node can refresh its selection of shortcuts randomly and independently from all other nodes.

A number of practical issues and implementation details can be addressed as follows. Note that the choice of the *shortcuts* at each node [as in step 3) of the contraction algorithm] is independent of both the source and destination nodes; in other words, *the same set of shortcuts* can be used to simultaneously accommodate the traffic for *any* source–destination pair. This is essential for extending the results to *multicast* in which the set of receivers is not known in advance.

B. Expansion Random Walk Algorithm

The ERWA is used to flow packets out of the source node S, as described below.

1) Expansion walk always starts from the source node S. During the expansion phase, only the position of the source node is required.
2) *Choosing a long-range link:* Every node v calculates its distance r_v to the source node s. It then chooses a random node u *uniformly* from the set of all nodes within distance $r_v = d(v,s)$ of the node itself.
3) *Deciding between a long-range versus a local link:* When node v receives a packet, if $d(u,s) > d(v,s)$ (where s is the source node), then v passes the packet to u. If $d(v,s) \geq d(u,s)$, then v passes the packet to a first neighbor on the lattice that is furthest from s.
4) Step 3) is repeated T times, where $T = O(\log N)$ is a prespecified integer value. The node in which the expansion phase ends is called the *relay node.*
5) *Refreshing the choice of long-range connection:* Each node can refresh its selection of shortcuts randomly and independently from all other nodes.

C. The Proposed Routing Protocol

For routing, an expansion phase starts from the source node and continues for $T = O(\log N)$ steps until it stops at a relay node v. A contraction walk then starts from v towards the destination. The extension to the case of multiple receivers (multicast scenario) is straightforward. To feed M receivers, simply, M contraction random walks start, in parallel, from the relay node v. If the routing algorithm can successfully accommodate a single contraction phase, it will also be able to accommodate multiple phases at the same time. Since the creation of the shortcuts at individual nodes in the contraction phase is independent of the position of the destination, the same set of links can be used to relay messages to multiple destinations in the contraction phase of a multicast.

IV. Analytical Performance Estimation

In this section, we calculate different performance metrics for our model network. Specifically, we are interested in calculating latency and average power consumption of routes across a large-scale network using the proposed algorithm. We state the results for a single receiver for clarity. The extension to multiple receivers is straightforward.

The main results of this section are summarized as follows.

- *Low latency:* The average communication latency is only $O(\log^2 N)$. This follows directly from Kleinberg's proof of the routability of small-world networks and will be discussed later on in this section.
- *Load balance:* The average power consumption of any node in the network at distance r_s from the source and r_d from the destination is bounded as
$$O(\log N \max\{r_d^{\alpha-2}\log r_d, r_s^{\alpha-2}\})$$
where α is the wireless path-loss exponent. The maximum average power consumed, per packet, by any node in the network is bounded as $O(N^{\alpha-2}\log^2 N)$.

A. Latency

We first note that the average latency of our algorithm is considerably less than $O(N)$ hops and, in fact, increases only polylogarithmically with network size, thus achieving scalable latency.

Proposition 4.1: For the defined network structure and routing scheme, the average latency is $O(\log^2 N)$. Also, there exists a constant $\kappa > 0$ independent of N, such that the probability that any routing path is longer than $\kappa \log^3 N$ is at most N^{-2}.

Proof: By construction, the expansion phase uses only $T = O(\log N)$ hops. The choice of the shortcuts in the contraction phase follows the small-world model introduced by Kleinberg in [5]. The polylogarithmic dependence of the latency on N is the direct result of Theorem 2 in [5]. We refer the interested reader to this paper for the full proof. In accordance with the terminology in [5], define the *phase* of the routing algorithm as follows: if the lattice distance of the node currently holding the packet from destination is between 2^j and 2^{j+1}, we say that the routing is in phase j. When the routing is in any phase j at any give node, the probability of going to the next phase in the next hop is bounded below by $c/\log N$ for some constant c, independent of the relay node and the phase. This follows from the long-range probability distribution $p(d) = \frac{\gamma}{\log N} d^{-2}$, where $\gamma > 0$ is a positive normalization constant and the factor $1/\log N$ is required to ensure the normalization of the probability distribution function on a grid of size $N \times N$. Please see [5, p. 8] for the proof.

Now, without loss of generality, assume that the destination is at the origin. Take a node at position (x, y) on the lattice, in phase j of the routing (i.e., $2^j < \max x, y \leq 2^{j+1}$). The probability that this node has a shortcut into a node in phase $j' < j$ is at least $p = \sum_{x',y':\max x',y'<j} \gamma/(|x - x'|^2 + |y - y'|^2)$, which can be lower bounded as $\gamma'/\log N$, for some γ' independent of x, y (see [5]).

Thus, in expected $O(\log N)$ steps, the packet will move to the next phase. There are a total of only $O(\log N)$ phases; therefore, the algorithm will reach the last phase (i.e., the packet reaches a node within $O(\log N)$ steps of the destination node) in expected $O(\log^2 N)$ hops.

The probability that the routing does not move to the next phase in K steps is at most

$$p_e(K) \leq (1 - c/\log N)^K \leq \exp\left(-cK/\log N\right).$$

Thus, the probability that the phase of routing is not increased by one after $4/c \log^2 N$ steps is

$$p_e\left(6/c \log^2 N\right) \leq \exp(-6 \log N) = N^{-6}.$$

Since there are at most N^2 nodes, and each routing path is at most N^2 long, the probability that there is any routing path in which the phase of the routing does not increase in $6/c \log^2 N$ steps will be at most $N^{-6} \times N^2 \times N^2 = N^{-2}$, which goes to zero as $N \rightarrow \infty$. Thus, for any routing path, the phase of the routing decreases by one in at most $O(\log^2 N)$ steps with probability that approaches one as $1 - N^{-2}$. Thus, the routing delay is bounded by $O(\log^3 N)$ hops with probability at least $1-N^{-2}$. ■

B. Markov Chain Modeling and Participation Probability

Our main goal is to characterize the average power consumption of nodes as a function of their relative position to the source or destination in either contraction or expansion phases. The calculation of this power requirement of a node depends on first calculating the probability with which a node will choose a shortcut (in the contraction phase) or a long-range (in the expansion phase) link when a packet arrives at it, and also the probability with which a node will be on a path. We calculate these probabilities using an Markov Chain (MC) model for our routing algorithms.

The stateless nature of our routing algorithm defines a Markov chain with $(N + 1)^2$ states for each packet transmission, where the state of the chain is identified by the index of the node currently holding the packet. The expected power consumption at a node i can therefore be defined as a reward function $P(i)$ on a slightly modified version of this Markov chain, as is explained shortly.

We divide the calculation of $P(i)$ into two steps: 1) calculating the probability that a node i is in the route of a random path and 2) calculating the average power consumed by i in relaying a received packet. $P(i)$, can then be calculated by multiplying these two quantities. In this section, we will calculate the first part, which we call the participation probability, that is, the probability that a node i participates in a random route. Since the expansion phase is regardless of the destination and the contraction is independent of the source, we can analyze each phase separately and then add them together to get the overall power consumption.

We start by calculating the participation probability for the contraction phase. We only concern ourselves with symmetric initial conditions, and therefore we only need to find the probability that a node at distance r from the destination will participate in a given route. Later on, we will consider the distribution of the range of connections to which a node at distance r from the destination will relay a received packet.

Results in this section, derived using a continuous approximation, show that the probability that a node at distance r from the destination participates in a contraction random walk is almost proportional to $\log r/r^2$ for moderately large r. Similar results hold for the case of expansion random walk; the probability that an expansion walk passes through a node at distance r from the source scales approximately as $1/r^2$, as is briefly considered in the Appendix.

Define the *participation probability distribution* vector ψ as a vector of length $(N+1)^2$, where $\psi(i)$ represents the probability that the ith node on the lattice participates in a random route. Our goal in the next section is to find (or at least upper bound for) this probability.

C. Approximating the Participation Probability

Let Π be the $(N + 1)^2 \times (N + 1)^2$ matrix of the transition probabilities for the MC in the previous section, i.e., $\Pi(i, j)$ is the probability that the routing currently in node i will move to node j in the next step. This initial Markov chain model has only one recurrent state, the origin or the destination node t; that is, all packets will reach the destination within an average time of $O(\log^2 N)$, regardless of their initial position in the network, and it is the only absorbing state. However, we want an MC that will model the flow of data, where new routes are initiated in a random manner and each node in the grid has a steady-state probability of being visited by one of these messages. In order to model this flow of data, we modify the Markov chain as follows. We allow transitions from the destination node (i.e., node labeled as t) and back to the network, i.e., we set $\Pi(t, b)$ to be equal to the probability that a new route starts from a bootstrap node b.

This modification makes all states recurrent and allows us to take into account the effect of initial conditions, i.e., how initial tokens for new routes are introduced in the network. For instance, if the initial node is chosen randomly from all nodes in the network, then $\Pi(t, b) = (N+1)^{-2}$ for all b. In our analysis, however, we will only consider the case where $\Pi(t, b) = 0, \forall b : d(t, b) < r_{\max}$ for some $r_{\max} \sim N$. In other words, all routes will start from a node far away from the destination. This is consistent with our expansion phase algorithm, where the tokens are routed far away from the source and reach relay nodes that can be assumed to be at the periphery of the grid.

With this initial condition, all nodes with distance $r_{\max}$ or less to the destination will become recurrent states of the chain. The steady-state probability distribution of this modified chain, denoted by the vector ϕ, will therefore indicate the *frequency* with which a packet will pass thorough a particular node.

The following lemma upper bounds the participation probability $\psi(i)$ as a function of $\phi(i)$.

Lemma 4.2: $\psi(i) \leq \kappa'' \phi(i) \log^2 N$ for some constant $\kappa'' > 0$.

Proof: We start by expanding $\phi(i)$ as follows:

$$\phi(i) = \sum_{p \in \mathcal{P}} \text{Prob}(p) \times I(i \in p)/L(p) \tag{1}$$

where $\mathcal{P}$ is the set of all routing paths, $L(p)$ is the length of path p, and $I(i \in p)$ is an indicator function that is equal to one if i is in path p and is zero otherwise.

From Proposition 4.1, we know that the probability that there exists any path p of length more than $\kappa \log^3 N$ is at most N^{-2}. Thus, with probability at least $1-N^{-2}$, we have

$$\phi(i) \geq \sum_{p \in \mathcal{P}} \text{Prob}(p) \times I(i \in p)/\kappa \log^3(N).$$

Therefore, we have

$$\psi(i) = \sum_{p \in \mathcal{P}} Prob(p) \times I(i \in p) \leq \phi(i) \kappa \log^3(N)$$

with probability at least $1-N^{-2}$.

This is, however, a rather conservative upper bound. The reason is that most of the path lengths are close to the average path length of $O(\log^2 N)$ rather than the maximum length of $O(\log^3 N)$. Thus, one may replace the random variable $L(p)$ in (1) with its average $\overline{L} = O(\log^2 N)$ (the so-called mean-field approximation). Our simulations indeed indicate that this approximation is valid and that

$$\psi(i) \leq \kappa''(\log^2 N)\phi(i) \tag{2}$$

for all i and an $\kappa'' > 0$ independent of i and N. ■

Thus to upper bound $\psi(i)$, in what follows, we continue to calculate the function $\phi(i)$. For the modified chain in steady state, the probability that a given node participates in a random hop of some random routing (indicated by the vector ϕ) has to satisfy the following consistency equation

$$\phi = \Pi \phi. \tag{3}$$

We therefore continue in solving for the steady-state probability distribution ϕ to prove the following theorem.

Theorem 4.3: Let (r_i, θ_i) be the polar coordinates of a node i with respect to the destination. Then $\phi(i) = \gamma \ln(r_i) r_i^{-2}$ for some constant γ.

In a hope to get a closed-form solution to (3), we adopt continuous approximations. This will allow us to rewrite the consistency (3) as an integral equation, the solution of which is only valid for r moderately large. To that end, we will need the following lemma, the proof of which is omitted for brevity.

Lemma 4.4: Take any disk C, centered at c with radius $R > N^{\xi}$, for some $0 < \xi < 1$ and a node v in the plane outside of C. Let $p(v, R)$ be the probability that v has a shortcut into C. Then, $p(v,R) = \frac{y}{\log N} \int_{c \in C} |v-c|^{-2} dc + O(\log^{-2} N)$. Furthermore, if v is on the perimeter of C, this probability converges to $p(R) = \frac{\log R}{\log N} z + O(\log^{-2} N)$, where constants z, y are independent of v, R, N.

Essentially, this states that the summation of the function $|v-c|^{-2}$ over a large enough disk converges to its integral with a relative error of no worse than $O(\log^{-1} N)$, and that the probability of finding a shortcut into a disk of radius R from any node on the perimeter of the disk scales as $\log(R)$. A similar result is in fact used by Kleinberg in the proof of routability of small-world networks in [5].

We also need the following simple geometric lemma.

Lemma 4.5: Take any node v and the set of its four neighbors. Assuming that a packet passes through v, it can only pass through at most two neighbors of v. Moreover, if the packet passes through exactly two neighbors of v, one of these neighbors is closer to the destination than v, and the other is further from the destination than v.

In a nutshell, this states that for every packet, there is at most one neighbor of any node v that might pass a packet to v through its local connection.

We are now ready to proceed in solving (3) as follows. First, note that the transition probability matrix Π can be written as the sum of three terms $\Pi = \Pi_{\text{local}} + \Pi_{\text{cut}} + \Pi_{\text{init}}$, where Π_{local} corresponds to the case where the packet currently at node j is sent to i through a local connection; therefore, $\Pi_{\text{local}}(j,i) = 0$ if $d(j,i) > 1$. Likewise, $\Pi_{\text{cut}}(j,i)$ corresponds to the case where the packet is sent from j to i through a shortcut. $\Pi_{\text{init}}(j,i)$, on the other hand, corresponds to the probability of initiating a new route at some node j, that is, $\Pi_{\text{init}}(j,i)$ is nonzero only if $j = t$ is the destination, in which case $\Pi(t,i)$ indicates the probability that a new route starts from a node i.

Thus

$$\phi(i) = \sum_j \phi(j)(\Pi_{\text{cut}}(j,i) + \Pi_{\text{local}}(j,i) + \Pi_{\text{init}}(j,i)). \tag{4}$$

Since the indexing of nodes is arbitrary, we index them by their polar coordinates, taking the destination as the origin, to get $\phi(r_i, \theta_i)$ as the probability that a node i at polar coordinates (r_i, θ_i) participates in a routing path

$$\begin{aligned}\Pi(r_i,\theta_i;r_j,\theta_j) &= \Pi_{\text{local}}(r_i,\theta_i;r_j,\theta_j) \\ &\quad + \Pi_{\text{cut}}(r_i,\theta_i;r_j,\theta_j) + \Pi_{\text{init}}(r_i,\theta_i;r_j,\theta_j)\end{aligned}$$

are defined similarly.

Now take any node v with coordinates r, θ such that $r > N^{\xi}$, for some $0 < \xi < 1$.

a) If a packet arrives at a neighbor of v and that neighbor node does not have a shortcut into the disk of radius $r-1$ around the target, it will have to pass the packet to v. Lemma 4.5 states that the number of such neighbor nodes is at most one. Let us call this node v^* and its coordinates r^*, θ^*.

b) v can always receive a packet through a shortcut from a node further than r to the target.

Let us assume that no node within a radius $r_{\max}$ of the destination is ever chosen as the initial node. In that case, $\Pi(r_o, \theta_o; r, \theta) = 0$ when $r_o < r$ *or* $r > r_{\max}$. We can then write the consistency (4), which is valid only for $r < r_{\max}$, as follows:

$$\phi(r,\theta) = \int_{r'=r+1}^{r_{\max}} \int_{\theta'=0}^{2\pi} \frac{y}{\ln(N)} \frac{\phi(r',\theta')}{r^2 + r'^2 - 2rr'\cos(\theta')} d\theta' dr' + \phi(r^*, \theta^*)(1 - z\ln(r^*)\ln^{-1} N) \tag{5}$$

where the first term comes from integrating the probability that a packet arrives into the node v through a shortcut from all other nodes at distance greater than or equal to r into v. We have also used Lemma 4.4 to change the summation into an integral, noting that $r > N^{\xi}$ for some $0 < \xi < 1$. The second term of the summation comes from the possible contribution of a single neighbor v^* that might send its packets locally to v (see Lemma 4.5), where $(1 - z\log(r^*)\ln^{-1} N)$ is the probability that v^* does not have a shortcut that takes the packet to a node closer to the origin than v.

Since the distance of v^* is much closer to v than r, we can approximate r^* by r and $\phi(r^*, \theta^*)$ by $\phi(r, \theta)$ itself to get

$$\begin{aligned}\phi(r,\theta) &\approx \frac{y}{\ln(r)} \int_{r'=r+1}^{r_{\max}} \frac{2r'dr'}{r'^2 - r^2} \\ &\quad \times \arctan\left(\frac{(r+r')\tan(\frac{\theta'}{2})}{r'-r}\right) \phi(r', \theta + \theta')|_{\theta'=0}^{\theta'=2\pi} \\ &= \frac{2y}{\ln(r)} \int_{r'=r+1}^{r_{\max}} \frac{\pi r'}{r'^2 - r^2} \phi(r') dr'. \end{aligned} \tag{6}$$

In getting the second line, we have assumed that the initial condition is symmetrical and, thus, $\phi(r, \theta)$ is independent of θ.

Equation (6) is a Volterra integral equation of the second kind, the solution of which we "guess" to be of the form

$$\phi(r) = \gamma \ln(r) r^{-2} \tag{7}$$

at least when $1 \ll r \ll r_{\max}$, where γ is a normalizing constant. To verify this, we insert it back into the right-hand side (RHS) of (6) and carry out the integration to get

$$\begin{aligned}\frac{\ln(r)}{2y}\mathrm{RHS} = &\gamma \frac{1}{2} r'^{-2} \left\{\log(r')^2 - \log\left((r'-r)/r\right)\log r\right\} \\ &+ \frac{1}{2} r^{-2} \Big\{ \mathrm{dilog}\left(r/r'\right) - \mathit{dilog}\left(r/(r'+r)\right) \\ &- \log(r')\log\left((r'+r)/r'\right) \Big\}\Big|_{r'=r+1}^{r_{\max}}\end{aligned}$$

where "dilog" is the dilogarithmic function defined as $\mathrm{dilog}(x) \triangleq \sum_{n=1}^{\infty} x^n/n^2$, when $|x| \leq 1$. Noting that $1 \ll r \ll r_{\max}$ and that $\mathrm{dilog}(x) < 1.65$ for all $x \leq 1$, the right-hand side will approximate to

$$\mathrm{RHS} \approx y\gamma \ln(r) r^{-2}$$

which is equal to $y\phi(r)$. If the constant $y = 1$, then $\phi(r) = \gamma \ln(r) r^{-2}$ is the actual solution for large r. Since the value of y is not exactly known, we cannot claim this to be the actual solution. Therefore, we have to rely on numerical simulations to verify that the actual form of the participation probability is indeed proportional to $\log(r) r^{-2}$ at least for large r.

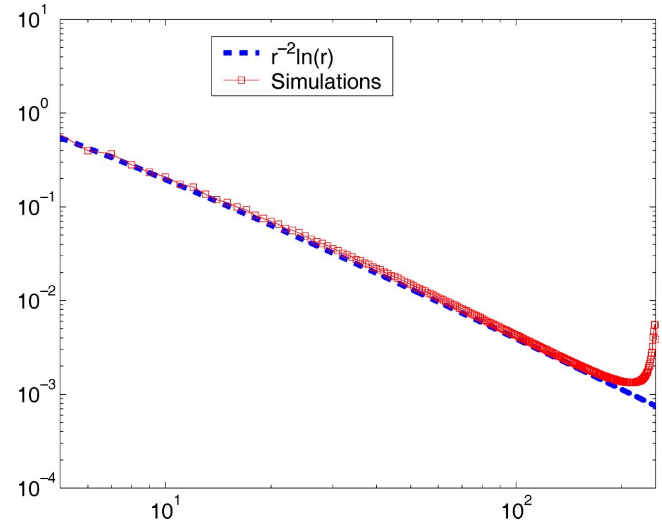

Fig. 1. The participation probability as a function of the normalized distance to the destination. Simulations are for a network of 500×500 and $r_{\max} = 250$. The excellent match to the theoretical prediction in (7) is evident.

One such set of simulations is reported in Fig. 1. In Section C of the Appendix, similar results are derived for the expansion phase; in particular, the probability that an expansion passes through a node at distance r_s from the source is shown to scale as $1/r_s^2$.

D. Power Distribution and Load Balance

The main milestone of this paper is to show that power consumption is essentially constant for all nodes, regardless of their distance to the source or destination. We compute the total power required for a single transmission between source and destination and also the average *per-node* power requirement. This will enable us to show that the proposed algorithm achieves scalable per-node power consumption by distributing the power requirement on the entire network, thus avoiding hot spots. Here is the main theorem proved in this section.

Theorem 4.6: In the contraction phase, the *per-node* power requirement for a node at distance r_d from the destination is $O\left(P_0 r_d^{\alpha-2}\log(r_d)\right)$. The *per-node* average power consumption in the expansion phase is $O(r_s^{\alpha-2}P_0)$, where r_s is the distance from the source.

We will prove this theorem in the rest of this section, using the results in previous sections. The theorem implies that for free-space communication (i.e., $\alpha = 2$), the power consumption is essentially uniformly distributed on all nodes inside the network in either traffic model.

We first consider the contraction phase. As stated before, the power consumed at a node v is a reward function defined on the underlying MC. To find the average reward at v, we have first calculated the probability that a random route passes through v. Now, assuming that a contraction route passes through v, we need to calculate the expected power v spent in routing the packet. Take a node v at distance r from the destination and assume that it has received a packet to be relayed towards the destination. The node v will use its shortcut for the relay if the

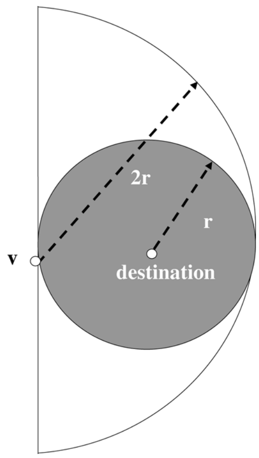

Fig. 2. Upper bounding the probability that a node v at distance r to the destination uses its shortcut connection to relay a message. This probability is upper bounded by the probability that it has any shortcut to the shaded disk, which in turn is upper bounded by the probability that it has a shortcut into the semicircle of radius $2r$ containing the disk of radius r.

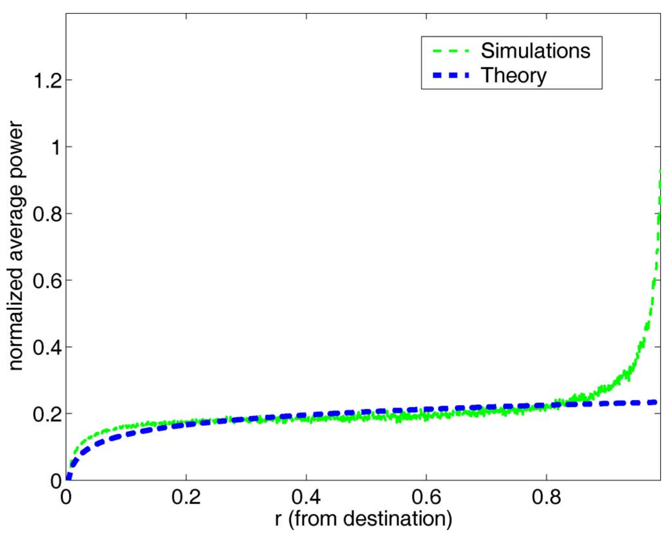

Fig. 3. Load balancing in contraction phase. Average power requirement for an entire network per packet as a function of distance to destination. The y-axis is normalized by $10^7 P_0$. N is 3000 and x-axis is normalized by 1500. Simulation results and analytical prediction found by multiplying $\phi(r)$ in (7) by r^2 are shown (constants are chosen to best fit the simulation data). Simulations are for 10^6 packet transmissions and are averaged over 50×50 meshes. Since the initial node is chosen uniformly from all nodes at normalized distance 1 from the destination, the average power in the simulations diverges at distance 1.

shortcut is to a node that is closer than r to the destination; i.e., the shortcut is to a node within a disk of radius r around the destination. This disk is contained in a half-disk of radius $2r$, originated at v, that is tangent to it (see Fig. 5). Let $P_{\text{avg}}(r)$ be the expected power required for a node at distance r from the destination to relay a received packet. $P_{\text{avg}}(r)$ is bounded as

$$\begin{aligned} P_{\text{avg}}(r) &\leq P_0 + h(\log N)^{-1} \int_{x=1}^{2r} (\pi x)(x^{-2})(P_0 x^{\alpha}) dx \\ &= P_0 + hP_0(\log N)^{-1} \int_{1}^{2r} x^{\alpha-1} dx \\ &\leq h' P_0 (\log N)^{-1} r^{\alpha} \end{aligned} \tag{8}$$

for some constant $h, h' > 0$ independent of N and r. In (8), P_0 is an upper bound on the power requirement assuming that no long-range connection is used for the communication. The summation is replaced with an integral, noting that the semicircle is large. In the integral, the first term (πx) corresponds to the arc length of a semicircle with radius x and x^{-2} corresponds to the probability that the node has a shortcut of length x at all. The last term is the power required to transmit the packet to a node at distance x.

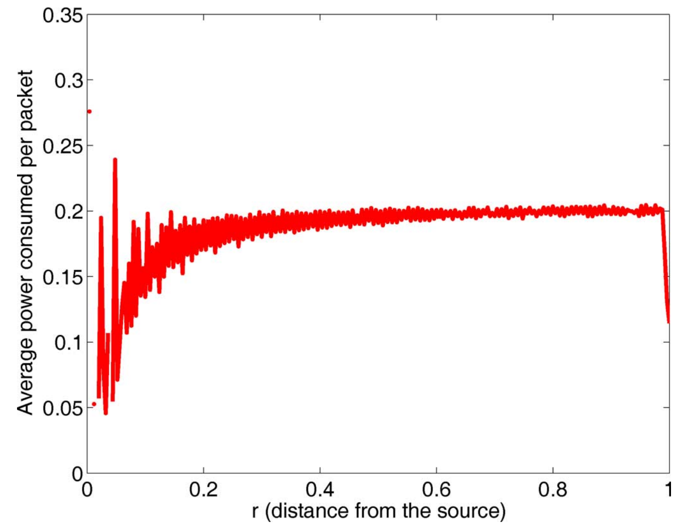

Fig. 4. Load balancing in the expansion phase. Average power requirement for entire network per packet, as a function of distance to source during the expansion phase. A total of 10^6 packets are transmitted in a network of size 1000×1000. All expansion walks start from the destination at position (500, 500) and continue 25 steps or when the distance to the destination becomes larger than 500. The distances (x-axis) are normalized by $N/2 = 500$, and the power is normalized by $10^7 P_0$.

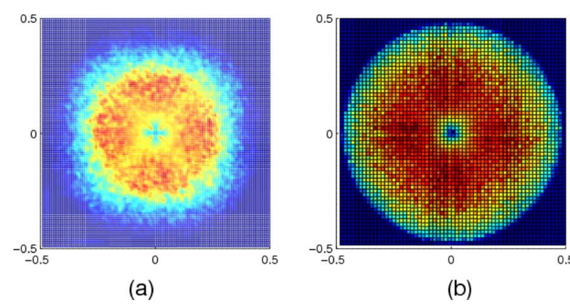

Fig. 5. The distribution of the load using (a) expansion only, with the source at origin, and (b) contraction only, with destination at origin. Simulations are for $N = 500$ and 1000, respectively, and all distances are normalized by $N/2$. The axes are shifted such that the source or destination is at the origin. For the contraction phase, the initial node is chosen uniformly at random from all nodes within normalized distance 0.5 from the destination.

Now recall that not all the nodes will receive a packet for relaying for each packet transmission. Rather, the probability that a random node at distance r receives a packet for a random transmission is given in (7). Now we are able to find $P_{\text{cont}}(r)$, the average power consumed by a node at distance r from the destination during the contraction phase. By invoking Lemma 4.2, (7), and (8), we get

$$\begin{aligned} P_{\text{cont}}(r) &= \psi(r) \times P_{\text{avg}}(r) \leq \kappa_1 \log^2 N \psi(r) \times P_{\text{avg}}(r) \\ &\leq \kappa_2 \log(N) P_0 r^{\alpha-2} \log(r) \end{aligned} \tag{9}$$

for moderately large r and constants κ_1, κ_2 independent of r and N.

The case of the expansion random walk is easier to handle. By the construction of the expansion random walk, the average length of the communication hop used by a node at distance r from the source is r (see Section III-B). Therefore, the average power consumed by a node at distance r from the source to relay a received packet in the expansion phase is at most $P_0 r^{\alpha}$. It is argued in Section B of the Appendix that the probability of participation in a random expansion walk for a node at distance r from the source scales as $1/r^2$. Thus, the overall average power consumed for expansion by a node at distance r from the source is

$$P_{\exp}(r) \le l r^{\alpha-2} P_0 \tag{10}$$

for some constant $l > 0$ independent of N, r.

V. Numerical Simulations

As discussed in the previous two sections, the algorithms introduced in this paper are simple and very easy to implement. We consider the following scenarios.

1) The contraction phase: communicating from random nodes in the network to a fixed destination. Our theoretical predictions for this part are that the average power consumption of a node at distance r from the destination scales as $\sim r^{\alpha-2}\log r$, where α is the path-loss exponent [see (9)]. For physical path loss ($\alpha = 2$), the power consumption is almost constant throughput the network (scales only logarithmically with r).
2) To verify the load balance during the expansion, we simulate a hypothetical scenario in which a number of expansion random walks start from a single source node. We show that the average power consumed by a node at distance r from the source during the expansion process scales as $\sim r^{\alpha-2}$ [see (10)]. Thus, for $\alpha = 2$, the power consumption is constant throughout the network.
3) We consider the overall distribution of the load resulting from expansion followed by contraction routing for a fixed source–destination pair. The approximate load balance during both the expansion and the contraction phases results in the balance of the overall load.
4) The proposed system can efficiently support multicasting (one to many) communication scenarios. We provide simulations for one such scenario as well. We verify the ability of our algorithm to simultaneously accommodate multiple streams while retaining the balance of the load.

Extensive simulations to verify the above claims are reported next. We have performed Monte Carlo simulations for a grid of $N+1$ by $N+1$ nodes for various values of N, using a path-loss exponent of $\alpha = 2$. The average power and participation activity are calculated for a total of $M = 10^6$ packet transmissions.

For contraction phase, Fig. 3 shows the average power consumption $P_{\text{cont}}(r)$ as a function of distance to destination. For comparison with theoretical derivations, Fig. 3 also plots $r^2\phi(r)$, the average power consumed by a node at distance r from the destination, with the participation probability given by (7). Please note that the initial node for the contraction walks in Fig. 3 is chosen uniformly from all nodes at normalized distance 1.0 from the destination. As such, the average power in the simulations diverges at distance 1.0. This divergence will be avoided if the initial node is chosen uniformly at random from all nodes within distance 1.

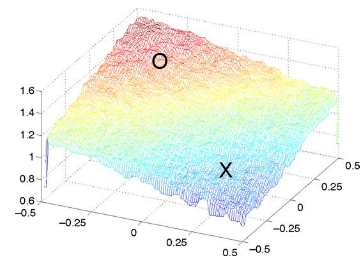

Fig. 6. Load balancing: average power consumption at all nodes in the network per transmitted packet. A source at a normalized position close to $(-0.25, -0.25)$ sends a stream of data to a node at a position close to (0.25, 0.25) using expansion followed by contraction.

Fig. 5 shows the distribution of the load over the entire network for expansion and contraction phases separately. In either case, the load is well balanced over all the network. In particular, no hot spots are created close to the source or destination. In all expansion scenarios reported, we have set the length of the expansion random walk T to 25.

While the analysis was done for a grid topology, simulation results reported next show that the results hold for random placement of nodes on the plane as well. In the rest of simulations, 1 000 000 nodes are randomly placed on a 1000×1000 square area. Each node has local connections to all nodes within a range of four units. The average number of local connections per node is around 50. It can be shown that a network with such local connectivity will be almost surely routable provided that the average number of local connections is at least $\Theta(\log N)$[18]. In our simulations, all packets were successfully routed, indicating the routability of the network.

The rest of the algorithm remains exactly the same. We have plotted the average load on the whole network resulting from communication of a single source–destination pair in Fig. 6 both when traffic is one way, that is, it always flows from the source to the destination, and when the traffic is symmetric, i.e., a packet is sent back from the destination to the source for each packet received.

Lastly, we have plotted the average power consumption over all nodes in the network in a multicast scenario in Fig. 7. The communication starts from the middle of the network (the origin) using an expansion random walk of length 25. Then, four contraction walks start from the relay node—one towards each of the four destinations. The position of the destinations is close to $(\pm 0.25, \pm 0.25)$ in the normalized coordinates. The load is excellently balanced on a very large plateau of the nodes in the middle of the network.

Various statistical properties for expansion and contraction phases with random node placement are reported in Table II in a network of size 1 000 000 for 10^6 packet communications.

VI. Concluding Remarks: Implementation Within Standard Wanet Routing Protocols

In this paper, we studied, within a rather theoretical setting, the concept of probabilistic use of long-range connections to

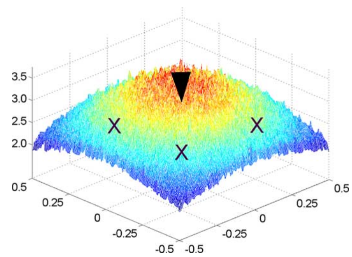

Fig. 7. Average power consumption for a multicast scenario. A source node at the origin sends a stream of packets to four nodes at positions (± 0.25, ± 0.25) through an expansion followed by four contraction random walks—one towards each destination node. The position of the source and three of the receivers are marked.

TABLE II
MEAN AND STANDARD DEVIATION (STD) OF THE AVERAGE POWER CONSUMPTION OF ALL NODES IN A NETWORK OF 1 000 000 NODES AND 10^7 CONSECUTIVE PACKET EMISSIONS FROM A SINGLE SOURCE NODE

Case	$Mean_P$	STD_P	$\frac{STD_P}{Mean_P}$	Latency
Contraction	1.82	2.14	1.17	23.4
Expansion	1.72	1.67	0.97	25.0
Expansion+Contraction	2.16	2.51	1.16	48.4

distribute the load and reduce the network latency in WANETs. We proved that distributed routing algorithms exist that can almost optimally take advantage of such long-range connections in the network. The local construction rules in our design suggest that nodes farther from source or destination have less chance of participating in the routing path while making longer connections with higher probability. This has the effect of distributing the average load almost uniformly on all nodes in the network. By choosing the distribution of the length of the connections according to a particular distribution, the routing paths are shown to be scalable.

The distributed nature of our algorithms allows for their full or partial implementation on top of existing, well-studied wireless ad hoc networking protocols. This will enable such protocols to work far more efficiently in applications close to the one discussed in this paper, i.e., where a long stream of data packets has to be communicated to a set of clients over multiple wireless hops. As an example, we briefly discuss how our algorithm can be easily implemented within the well-studied Dynamic Source Routing Protocol (DSR) [17]. DSR consists of two main mechanisms: (1) route discovery, which is used to create routing paths from source to destination, on demand, and 2) route maintenance, which is responsible for detecting the break of a source–destination path and creating alternative routes.

DSR does not specify the connectivity structure of the underlying wireless network. Nodes might use different algorithms and subprotocols to maintain their local or long-range connections. DSR uses a simple algorithm for route discovery. Put simply, to create a routing path from S to D, a route discovery message starts from S and is sent towards the destination. Each relay node appends its own identity (or address) to the message until it reaches the destination node D. This creates a path that will be communicated back to the source. Multiple alternative paths may be created between a source and its destination. During the route discovery, the choice of the next hop at a given relay node is left to the node itself. Nodes may use simple greedy strategies, for instance, and choose the node closest to destination as the next hop.

Our routing algorithm can be easily implemented in conjunction with DSR. One only needs to ensure that nodes retain long-range connections that are chosen according to the $1/d^2$ probability law. Now note that the choice of long-range connections in both the expansion and contraction phases are independent from the position of destination. Thus, these shortcuts can be maintained regardless of the position of the destination nodes, or their mobility. In our analysis, we assumed that shortcuts are made for each packet on the fly. This, however, is not necessary. In fact, a node may choose at least two shortcut contacts in an off-the-band process. When a shortcut is required, the node will choose only one of the two shortcuts randomly and assumes that the other shortcut does not exist.

Now consider the route discovery protocol in DSR. When a node receives a route discovery packet, it will decide if the packet is in expansion phase by checking the number of hops the packet has taken so far. If the packet is in expansion phase, the node will assume that its long-range connection corresponding to the expansion phase does not exist. Otherwise, if the number of hops indicates that the packet is in contraction phase, the node will assume that only one of its two long-range connections corresponding to the contraction phase is open. To create multiple paths, one would simply initiate the route discovery algorithms a number of times, as is also envisioned in DSR.

Lastly, while our analysis was based on the assumption that all nodes have access to long-range connections, in fact, only a small, random fraction of nodes are required to maintain long-range connections (see Section C in the Appendix).

APPENDIX

A. Bandwidth Requirement

Throughout this paper, we have neglected the interference of communicating nodes on each other. This assumption holds if the system bandwidth is high enough. We now show that a bandwidth of only $B = O(\log^3 N)$ is sufficient.

Let us assume that data with a rate λ is being communicate from the source to destination(s). Take a packet of length b starting from some source s towards a destination t. Let $L_{\max} = O(\log^3 N)$ be the *maximum* number of hops that a packet has to undertake. Let B be the total bandwidth of the system. The idea is to divide the bandwidth into $L_{\max}$ noninterfering channels, each of bandwidth $B = B/L_{\max}$, and let each of at most $L_{\max}$ communication hops take place in one of the these channels. Note that the choice of channels can be negotiated between the two communicating nodes based on the number of hops the packet has taken so far.

Since the communication rate is λ, the amount of data communicated at each channel is at most λ and is constant. Since there is no interference in either channel, the amount of bandwidth required for each channel is a constant W. The total bandwidth is thus $W \times L_{\max} = O(\log^3 N)$.

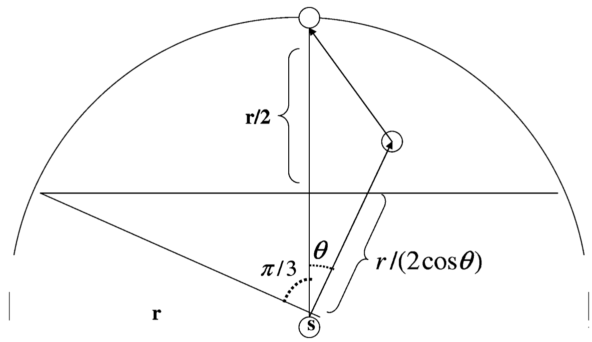

Fig. 8. Derivation of the integral (11). The consistency equation is written for a node v at distance r from the source s. v might receive a packet from all nodes u, with polar coordinate (r_u, θ) such that $-\pi/3 < \theta < \pi/3$ and $r/(2\cos(\theta)) < r_u < r$. For each of such nodes u, the node v will receive a packet with probability $\psi_{\exp}(r_u)(\pi r_u^2)^{-1}$.

B. Participation Probability in Expansion Phase

The participation probability for the case of contraction random walk was derived in detail in the text. In this section of the Appendix, we briefly consider the participation probability for the case of the expansion random walk. We show that, similar to the contraction scenario $\psi_{\exp}(r)$, the probability that a random route passes through a node at distance r from the source scales as $1/r^2$.

From the description of the expansion random walk, it is clear that during the expansion phase, a node v might receive a packet through a shortcut from a node v' only when $d(v', s) \geq d(v, v')$, where s is the source node. Just as in the contraction scenario (see Lemma 4.5), a node might also receive a packet from at most one of its immediate neighbors that is closer to the source than v. Let $r^* \approx r$ be the distance of this neighbor to the source and $w(r^*)$ be the probability that a packet is received through a shortcut from v^* if the route actually passes through v^*.

A consistency equation for $\psi_{\exp}(r)$ can be written as follows (see Fig. 8 for explanation):

$$\psi_{\exp}(r) = \int_{\theta=-\pi/3}^{\pi/3} \int_{r=r/(2\cos(\theta))}^{r} \frac{\psi_{\exp}(r')}{\pi r'^2} r' dr' d\theta + w(r^*)\psi_{\exp}(r^*). \tag{11}$$

For large r, $w(r^*)$ will converge to 1/2 and $\psi(r^*)$ to $\psi(r)$, resulting in

$$\psi_{\exp}(r) = 2\int_{\theta=-\pi/3}^{\pi/3} \int_{r=r/(2\cos(\theta))}^{r} \frac{\psi_{\exp}(r')}{\pi r'^2} r' dr' d\theta \tag{12}$$

at least when r is large.

This equation, in many ways, resembles the integral (6). While we are unable to solve this equation exactly, we note that a function $\psi_{\exp}(r) \propto r^{-2}$ provides a consistent functional form to (12). We rely on simulations to conclude that the actual solution indeed behaves as $1/r^2$. Using $\psi_{\exp}(r) = r^{-2}$, the right-hand side of (12) can be worked out, resulting in

$$\begin{aligned}\text{RHS} &= 2l\int_{\theta=-\pi/3}^{\pi/3} \int_{r=r/(2\cos(\theta))}^{r} \frac{1}{\pi r'^3} dr' d\theta \\ &= 2lr^{-2}\int_{\theta=-\pi/3}^{\pi/3} (2\pi)^{-1}(4\cos(\theta)^2 - 1) d\theta \\ &= r^{-2}\frac{(\sin(2\pi/3) + 2\pi/3)}{2\pi} \approx 0.94lr^{-2} \approx \psi_{\exp}(r).\end{aligned}$$

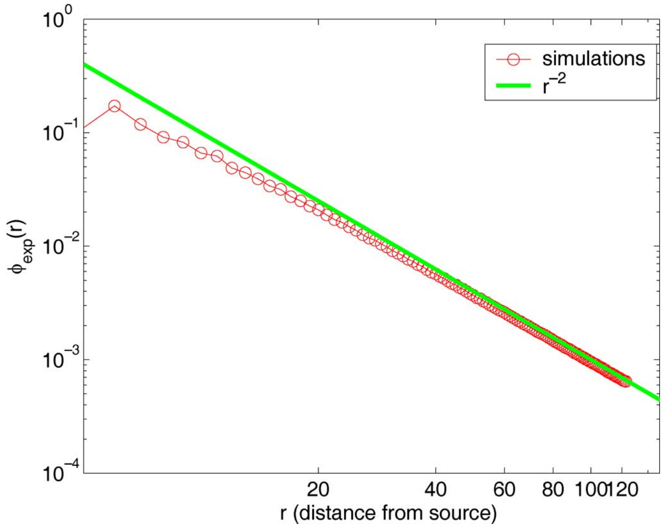

Fig. 9. Participation probability for the expansion phase as a function of the distance to the source. $N = 1000$, and results are averaged over 10^6 packet transmissions. A function of form $1/r^2$ provides an excellent fit to the actual participation probability when r is more than 30 and an upper bound for lower values of r.

Simulation results in Fig. 9 show that the actual participation probability of the expansion phase very closely follows a $1/r^2$ relation at moderately large r values.

C. Utilizing Power Heterogeneity

In developments in Section III, we assumed that each node is capable of creating shortcuts. In other words, we assumed that all nodes have the capability of connecting to nodes that are arbitrarily far away in the network. Our analysis showed that by selecting the links appropriately, the average power consumption for every node can be made essentially balanced.

In some applications, however, the power capability of nodes is heterogeneous; only a fraction of nodes are capable of making long range connections. Such nodes are sometimes called *leader* or *dominating nodes*[3], [8].

The structures and algorithms in Section III can be modified to impose a hierarchical structure that utilizes the existence of such dominating nodes. In the proposed structure, nodes can belong to $\log N$ different categories based on their power resources. How nodes are assigned to categories is irrelevant. However, the routing algorithm will be successful if a certain number of nodes belong to each of the categories.

We divide shortcuts into $\log N$ ranges. Let $\tau(v, k)$ be the set of all nodes v' such that $2^{k-1} < d(v, v') \leq 2^k$. A node v in category k selects a single shortcut uniformly randomly from all nodes in the set $\tau(v, k)$. Each node will have its usual four nearest neighbor connections on the grid. The routing remains a greedy algorithm. We now show that if nodes randomly belong to each of the $\log N$ categories, the average routing latency remains $O(\log^2 N)$. We need the following lemma first.

Lemma 1.1: Let $k \geq 4$. Take two nodes i, t such that $t \in \tau(i, k)$. If node i is of category k, then, with probability at least 0.15, the node i has a shortcut into a node v such that $t \in \tau(v, k-1)$

Proof: It suffices to prove the lemma assuming $d(i, t) = 2^k$. The area of points that are within the distance $d/2$ from

t and distance d from i can be calculated, through somewhat elementary calculations, to be

$$S = d^2 \left(\arccos(1/4) + \arccos(3/4)/4 - \sqrt{5}/4 \right) \geq 0.6d^2.$$

The area between two disks with radii $d/2$ and d centered at i is $3\pi d^2/4$. Therefore, the probability that the node i has a shortcut into a node in the next phase is at least $= 0.6d^2/(3\pi/4) > 0.15$, which proves the lemma. ■

With the aid of the above lemma, the following proposition can be proved about the routing properties of the modified contraction phase.

Proposition 1.2: Suppose that nodes randomly belong to each of the $\log N$ categories. If the shortcuts are selected with the algorithm in this Appendix, the contraction phase of Section III has an average latency of $O(\log^2 N)$.

Proof: Kleinberg's proof of Proposition 4.1 can be modified to prove the current proposition. Let us consider a packet destined for a node t. Let v be the current node holding the packet. We say that the routing is in phase j if $j \in \tau(t, j)$. If the routing is in phase $j < 4$, then the routing will end using local connections in at most four steps. Else, if $j \geq 4$ and the current node is of category j, then, from Lemma 1.1, it will have a shortcut into a node in phase $j-1$ of the target with probability at least 0.15. If not, the packet will follow the connection that takes it closest to the target. It will take, on average, at most $\log N$ nodes to visit before a node in category j can be found. After that, the phase of the routing will decrease by one with probability of at least 0.15. Therefore, on average, it takes at most $\log N/0.15 < 7 \log N$ steps to decrease the phase of the routing by at least one. Since there are only $\log N$ phases, it takes, on average, at most $7 \log^2 N$ steps for the routing to end. ■

With the same methods in this paper, it can be shown that the power load on *all nodes within the same category* is almost balanced (within a factor of $\log N$), regardless of their distance to the destination. The power load, however, increases exponentially for nodes in higher categories, thus allowing more powerful nodes to carry most of the power requirement required for low-latency communication.

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Behnam A. Rezaei received the B.Sc. degree from Sharif University of Technology, Tehran, Iran, in 2000 and the M.Sc. degree from the University of California, Los Angeles (UCLA), in 2002, both in electrical engineering. He is currently pursuing the Ph.D. degree at UCLA.

His research interests include large-scale network dynamics and P2P computing.

Nima Sarshar received the B.Sc. degree from Sharif University of Technology, Tehran, Iran, in 2000, the M.Sc. degree from the University of California, Los Angeles, in 2002, and the Ph.D. degree from McMaster University, Hamilton, ON, Canada, in 2006, all in electrical engineering.

He is currently an Assistant Professor with the Faculty of Engineering, University of Regina, Regina, SK, Canada.

Vwani P. Roychowdhury received the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, in 1989.

From 1991 to 1996, he was a Faculty Member with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, where he became an Associate Professor in 1995. In 1996, he joined the University of California, Los Angeles, where he is currently a Professor of electrical engineering.